How Many Hours Are in a Simulated Day?
The Effects of Time Endowment on the Results of Tax-Policy Simulation Models

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I. Introduction

Simulation models have often been used to assess the effects of tax-policy changes on labor supply. This can be done in a variety of ways. One of the most popular methods is to specify a utility function, defined over consumption and leisure.[1] When this type of utility function is used, the modeler must choose the total endowment of time, which the model consumer may allocate between consumption and leisure. I define the "time-endowment ratio," Φ, as

$$(1) \qquad \Phi = E/H,$$

where E is the consumer's endowment of time, and H is the amount of labor that is supplied in the base case.

Researchers have used an extraordinary variety of values for Φ. For example, Ballard, Fullerton, Shoven, and Whalley (1985, p. 135) choose a value of 1.75 for Φ, "...to reflect that individuals typically work a forty-hour [week], out of a possible seventy-hour week." Jensen and Rutherford (1999) set ø=1.5 Fullerton and Rogers (1993) specify a time endowment of

[1] Among the many simulation modelers who have adopted this approach are Auerbach and Kotlikoff (1987), Ballard, Fullerton, Shoven, and Whalley (1985), Fullerton and Rogers (1993), and Jorgenson and Wilcoxen (1998). Of course, it is possible to specify a labor/leisure choice without having leisure as an argument of the utility function. Utility could be specified as depending on the number of hours worked. In this setting, some of the problems discussed in this paper would not arise. However, the practice of having leisure as an argument of the utility function is very widespread.

4000 hours per year, which yields a value of Φ of about 2.0 for several of their consumer groups. Auerbach and Kotlikoff (1987, p. 52) set Φ to 2.5, to represent "...a full-time labor endowment of 5000 hours per year [in which workers are assumed to] work 2000 hours per year, or 40 hours per week." Greenwood and Huffman (1991, p. 175) use a value of approximately 3.846 for Φ: "This number corresponds to the average ratio of total hours worked to total nonsleeping hours of the working age population observed in the U.S. data." In the model of Jorgenson and Willxcoxen (1998), the average value of ø is about 4.1.[2] Mendoza and Tesar (1998) set Φ equal to 5.0.

When they discuss the choice of Φ, the authors of these studies often make it sound as if the main purpose of Φ (or even its sole purpose) is to specify the amount of time available. There is usually little or no emphasis on any other effects that Φ might have on the model. Most researchers perform little or no sensitivity analysis with respect to this parameter.[3]

Moreover, there is inevitably a considerable amount of arbitrariness in choosing Φ in this way. Should we say that eight hours per day are necessary for sleep, leaving 16 hours per day available for work? Or should we subtract a few hours for eating breakfast, taking a shower, paying bills, cleaning house, etc., which might leave only 14 or 12 or 10 hours per day for work? Should we allow for weekends and holidays? Do we make different choices for those younger than 20, or 18, or 16 years old? Do we make different choices for mothers with small children? Do we make different choices for the elderly? If so, at what age does one become elderly?

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[2] This value for ø was reported in personal communication with P.J. Wilcoxen, October 4, 1999.

[3] The literature does contain a few papers in which authors perform sensitivity analysis with respect to the time-endowment parameter. For example, see Fullerton, Henderson, and Shoven (1984), whose results will be discussed below. Mendoza and Tesar (1998) compare the case of $\Phi = 5$ with the case of $\Phi = 1$, in which case labor supply is perfectly inelastic.

This arbitrariness would be of little importance, if it were true that Φ had little effect on the results from simulation models. In fact, however, the purpose of this paper is to demonstrate that Φ can often have a very substantial effect on the results from tax-policy simulation models. In a static model, if we hold constant the uncompensated labor-supply elasticity, the time endowment determines the total-income elasticity of labor supply and the compensated elasticity. This can have significant effects on welfare calculations. In a dynamic model, for a policy change that increases the rate of return, consumers will desire to decrease their consumption of both current goods and current leisure. In other words, in addition to wanting to consume less, they will want to increase their labor supply. The time-endowment parameter plays a powerful role in controlling this effect.

If Φ is actually important, then how do we choose the "correct" value of Φ? The passages quoted above would seem to suggest that meaningful answers can be found by thinking about the number of hours in a day. This is unfortunate. A better approach would be merely to treat Φ as a parameter. Once we do this, we can concentrate on using Φ to control the elasticities in the model, which is far more important than using Φ to worry about the length of the day.[4]

In this paper, I will show the effects of Φ, using modified versions of several models that I have used previously. In the next section, I will demonstrate the effects of Φ in static models, including both a one-consumer model and a multi-consumer model. In section 3, I will demonstrate the effects of Φ in an infinite-horizon model. In section 4, I consider an overlapping-generations life-cycle model. Section 5 is a brief conclusion.

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[4] In rare cases, researchers are interested in the actual number of hours devoted to a variety of activities so that the actual amount of leisure is important. For example, see Harrison, Lau, and Williams (1999). However, in the vast majority of tax-policy simulation models, the labor-supply elasticity is much more important than the number of hours devoted to leisure.

II. The Effects of the Time Endowment in Static Models

A. A One-Consumer Model

It is common for researchers to model the static labor-supply choice by using a constant-elasticity-of-substitution (C.E.S.) utility function, defined over leisure, ℓ, and consumption of goods, C. The consumer's utility function takes the form

$$(2) \qquad U = \left[\beta^{\frac{1}{\varepsilon}} \ell^{\frac{\varepsilon-1}{\varepsilon}} + (1-\beta)^{\frac{1}{\varepsilon}} C^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where ε is the elasticity of substitution and β is a weighting parameter. The budget constraint states that the net-of-tax value of the consumer's endowment of labor, plus the net value of non-labor income, should equal expenditure on goods and leisure. Thus,

$$(3) \qquad \mathrm{W}\,\mathrm{E} + Y = \mathrm{W}\ell + \mathrm{P}C,$$

where W is the net-of-tax wage rate, E is the time endowment, *Y* is non-labor income, and P is the price of consumption goods. In this context, non-labor income includes both transfer payments and capital income. The time endowment is the sum of the amounts of labor and leisure: E = H $+ \ell$.

The researcher's goal is to calibrate the parameters of the utility function and the budget constraint, so as to replicate the benchmark data, and at the same time impose the desired elasticities on the model. The first step in this process is to choose a value of the static uncompensated labor-supply elasticity, based on the econometric literature.[5] In most of my work with static models, I have chosen uncompensated elasticities that are fairly close to zero (and sometimes negative) for males, and positive for females. My view is that the weighted average of the uncompensated labor-supply elasticities for the entire economy is probably in the range from zero to 0.15, although sensitivity analysis is certainly appropriate.

The next steps involve deriving expressions that connect the parameters of the utility function and the budget constraint with the exogenously specified value of the uncompensated labor-supply elasticity. First, we use standard techniques to derive the leisure-demand function:

$$(4) \qquad \ell = \frac{\beta I}{W^{\varepsilon}\left[\beta W^{1-\varepsilon} + (1-\beta)P^{1-\varepsilon}\right]},$$

where I = WE + Y is the consumer's "full income."

The uncompensated leisure-demand elasticity, η_{ℓ}, is

$$(5) \qquad \eta_{\ell} = \frac{\partial \ell}{\partial W}\frac{W}{\ell} = \frac{\beta EW}{W^{\varepsilon}\Delta\ell} - \frac{\beta(1-\varepsilon)W}{W^{\varepsilon}\Delta} - \varepsilon,$$

where $\Delta = \beta W^{1-\varepsilon} + (1-\beta)P^{1-\varepsilon}$. The uncompensated labor-supply elasticity, η_L, can be expressed in terms the amount of labor supplied, H, or in terms of the amount of leisure, ℓ:

[5] For summaries of this literature, see Burtless (1987), Heckman (1993), and Killingsworth (1983).

$$\text{(6)} \qquad \eta_L = \frac{\partial H}{\partial W}\frac{W}{H} = \frac{\partial(E-\ell)}{\partial W}\frac{W}{(E-\ell)} = \left(\frac{\partial E}{\partial W} - \frac{\partial \ell}{\partial W}\right)\frac{W}{(E-\ell)}.$$

Manipulating equations (5) and (6), and using the fact that $\partial E/\partial W = 0$, we derive an expression for the leisure-demand elasticity, η_ℓ, in terms of the uncompensated labor-supply elasticity, η_L, and the time-endowment parameter, Φ:

$$\text{(7)} \qquad \eta_\ell = -\eta_L \frac{(E-\ell)}{\ell} = -\eta_L\left(\frac{1}{\Phi - 1}\right).$$

Next, we use equations (5) and (7) to solve for ε, the elasticity of substitution between consumption and leisure, in terms of the time endowment and the labor-supply elasticity. Once we have solved for ε, we can then solve for β, the weighting parameter in the utility function.

The procedure described here can be used to calibrate the model very precisely to a desired value of the uncompensated labor-supply elasticity. This is important, since the uncompensated elasticity has an important effect on the results in a simulation model. However, the *compensated* labor-supply elasticity also plays a prominent role in many simulations.[6] By the Slutsky decomposition, we know that the difference between the compensated and uncompensated elasticities will be equal to the absolute value of the "total-income elasticity" of

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[6] Ballard (1990a) distinguishes between differential analyses, in which the compensated elasticities are most important, and balanced-budget analyses , in which the uncompensated elasticities are most important. The compensated elasticity is not crucial in *all* simulation experiments, but it often plays a crucial role.

labor supply. As in any Slutsky decomposition, the total-income elasticity (denoted η_I) will depend on the budget shares. In this case, the absolute value of η_I will increase monotonically with the time-endowment parameter, Φ.

Specifically, our goal is to derive the compensated labor-supply elasticity in terms of prices and preference parameters. We begin by using equation (4), and a similar equation for the demand for goods, to calculate the expenditure function, EX:

$$(8)\quad E^* = V\left[\beta W^{1-\varepsilon} + (1-\beta)P^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}.$$

In equation (8), V is indirect utility. The equation shows the amount of income that is necessary to achieve a given level of utility, as a function of prices and preference parameters.

Shephard's Lemma tells us that the compensated leisure-demand function is the derivative of the expenditure function with respect to the wage rate:

$$(9)\quad \frac{\partial E^*}{\partial W} = \ell^* = V\beta W^{-\varepsilon}\left[\beta W^{1-\varepsilon} + (1-\beta)P^{1-\varepsilon}\right]^{\frac{\varepsilon}{1-\varepsilon}}.$$

We differentiate equation (9) with respect to the wage rate, to get the Slutsky derivative:

$$\frac{\partial \ell^*}{\partial W} = YB\left\{\varepsilon\beta W^{-2\varepsilon}\Delta^{\frac{2\varepsilon-1}{1-\varepsilon}} - \varepsilon W^{-\varepsilon-1}\Delta^{\frac{\varepsilon}{1-\varepsilon}}\right\},$$

(10)

where $\Delta = \beta W^{1-\varepsilon} + (1-B)P^{1-E}$.

Multiplying by $\frac{W}{\ell *}$ (and using equation (9)) gives us the compensated leisure-demand elasticity, η_{ℓ}^{*}:

$$(11)\quad \eta_{\ell}^{*} = \frac{\varepsilon W^{\varepsilon+1}\left\{\beta W^{-2\varepsilon}\,\Delta^{\frac{2\varepsilon-1}{1-\varepsilon}} - W^{-\varepsilon-1}\,\Delta^{\frac{\varepsilon}{1-\varepsilon}}\right\}}{\Delta^{\frac{\varepsilon}{1-\varepsilon}}}$$

Equation (11) is the compensated leisure-demand elasticity, but we need the compensated *labor-supply* elasticity. To make the conversion, we use a relationship that is analogous to equation (7):

$$(12)\quad \eta_{L}^{*} = (1-0)\eta_{\ell}*,$$

where η_{L}^{*} is the compensated labor-supply elasticity. If we substitute equation (11) into equation (12), we get an expression for the compensated labor-supply elasticity in terms of prices, utility-function parameters, and the time-endowment ratio:

$$(13)\quad \eta_{\ell}^{*} = \frac{(1-\Phi)\varepsilon W^{\varepsilon+1}\left\{\beta W^{-2\varepsilon}\,\Delta^{\frac{2\varepsilon-1}{1-\varepsilon}} - W^{-\varepsilon-1}\,\Delta^{\frac{\varepsilon}{1-\varepsilon}}\right\}}{\Delta^{\frac{\varepsilon}{1-\varepsilon}}}.$$

The $(1-\Phi)$ term must always be negative. The rest of equation (13) is also negative, so that the combined expression is positive, as required by theory. The compensated labor-demand elasticity is increasing in Φ, the time-endowment parameter.

The model used in Ballard (1990a) is well-suited for exploring the sensitivity of the results with respect to Φ. This is a static model, with only a single consumer. The consumer who is assumed to behave according to the C.E.S. utility function shown above. The model has two production sectors. In Ballard (1990a), the outputs of the two production sectors were sometimes subjected to different sales tax rates. However, in this paper, the sales tax rates will be set equal to zero, so that we may focus exclusively on the labor-supply effects.

The econometric literature has produced a range of estimates of the total-income elasticity of labor supply, η_I, but most of them are not exceptionally large in absolute value. Based on my reading of the literature, a value of -0.1 would not be unreasonable. In the model of Ballard (1990a), when the initial labor tax rate is 40%, this value of η_I is implied by a value of 1.213 for Φ. Thus, the value of Φ that is necessary to produce a very reasonable value for η_I is far lower than virtually all of the values of Φ that have been chosen arbitrarily in the simulation literature.

If we set $\Phi = 2.5$ (the value chosen by Auerbach and Kotlikoff), the implied value of η_I is -0.4421, which is far larger than most of the econometric estimates of the total-income elasticity. If we set $\Phi = 5.0$ (chosen by Mendoza and Tesar), the implied value of η_I is -0.6787.

In Table 1, we show the effects of Φ (and therefore η_I) on the welfare effects from a simple tax-policy experiment, using the model of Ballard (1990a). We begin with a labor tax rate of 40%, and an uncompensated labor-supply elasticity of 0.1, and then we replace the labor taxes with a lump-sum tax that yields the same amount of tax revenue for the government.[7] When we replace a distortionary tax with a lump-sum tax, the consumer's welfare is improved by more than the amount of tax revenue that is shifted from a labor tax to a lump-sum tax. The average excess burden of this change is defined as

$$(14) \qquad \text{Average Excess Burden} = -(EV / \Delta R) - 1,$$

where EV is the change in consumer welfare from replacing a labor tax with a lump-sum tax of equal revenue yield, as measured by the equivalent variation, and ΔR is the amount of revenue replaced.

Table 1 shows that the average excess burden of the labor tax depends strongly on the value of η_I, and therefore also depends on Φ. When $\Phi = 1.213$, so that $\eta_I = -0.1$, the average excess burden is approximately 12.9%. When $\Phi = 5.0$, so that $\eta_I = -0.679$, the average excess burden is approximately 38.9%. In my view, the 12.9% estimate of the average excess burden is reasonable, because it is based on a reasonable value of η_I, while the 38.9% estimate could be a very misleading guide to policy.

Thus, the researcher is presented with a choice. One strategy is to select a value for Φ, based on an essentially arbitrary assumption about the number of hours available. This approach can lead to excessive income elasticities of labor supply, and therefore to excessive estimates of excess burden. The second strategy is to choose the desired value of the total-income elasticity

[7] This model, as well as the other models on which we report here, is solved using the tâ tonnement algorithm developed by Kimbell and Harrison (1986).

of labor supply, and to solve for the value of Φ that is consistent with that elasticity. In my opinion, the latter strategy is far superior.

B. A Multi-Consumer Model

In the preceding sub-section, we investigated a one-consumer model. By its very nature, such a model is incapable of exploring the distributional implications of tax-policy changes. Here, we consider a multi-consumer model, which has been used to investigate the efficiency and redistributive effects of tax/transfer policies.

Ballard (1988) and Ballard and Goddeeris (1996) report the results from multi-consumer simulation models that are otherwise fairly similar to the one-consumer model of Ballard (1990a). In this section, we focus on the sensitivity of the results from Ballard and Goddeeris (1996). This model has 816 consumer groups, representing the U.S. population for 1979.[8] The groups are distinguished on the basis of income class, gender of household head, number of persons in the household, and labor-force-participation status of the household head (and of the spouse of the household head, in the case of married couples).[9] Ballard and Goddeeris allow for the possibility that the different consumer groups may have different labor-supply elasticities. Their "central-case" uncompensated elasticities are -0.05 for men, 0.20 for women, and 0.10 for

[8] Data for 1979 are used, in order to make the results as comparable as possible with the results of Ballard (1988). In a further extension of these methods, Ballard and Goddeeris (1999) use data for 1991 to create a data set with 523 consumer groups, which is used to assess the effects of proposals for universal health-insurance coverage in the United States.

[9] The calibration techniques used for the consumer groups with one worker are identical to those described in Section I-A, above. For married couples in which both spouses are in the labor force, Ballard and Goddeeris adopt a utility function in which consumption and the leisure of *both* workers are arguments. This requires the specification of a separate Φ parameter for each spouse. The calibration techniques are more complicated than those described in Section I-A of this paper, but the basic idea is the same. For more details, see Ballard (1987).

married couples, along with a total-income elasticity of -0.2 for every group. The weighted average of these uncompensated elasticities is about 0.04. Because there are a number of differences among the various consumer groups in the Ballard-Goddeeris model, the value of Φ that is necessary to generate a total-income elasticity of -0.2 differs from group to group. However, for many of the groups, the required value of Φ is in the vicinity of 1.4.

The main concern of Ballard and Goddeeris is to calculate the "marginal efficiency cost of redistribution" (MECR), which is designed to summarize the tradeoff between equality and efficiency, associated with a redistributive change in tax/transfer policy. The MECR is defined as

$$(15) \quad MECR = - \frac{\Sigma \text{ EV for welfare losers}}{\Sigma \text{ EV for welfare winners}} - 1.$$

For the labor-supply elasticities mentioned in the preceding paragraph, for a demogrant financed by a proportional increase in labor tax rates from a base-case value of 40%, the MECR is 41.5%. In other words, the sum of the losses to the higher-income households that lose from the policy change is 41.5% larger than the sum of the gains for the lower-income households that gain from the policy change.[10]

As mentioned above, these results are based on values of Φ that are often in the vicinity of 1.4. What will happen when we change Φ (while holding the uncompensated elasticities constant)? When $\Phi = 2.5$ for every consumer group, the weighted average of the implied total-income elasticities of labor supply increases all the way to -0.517. This is far beyond the range of most econometric estimates of the total-income elasticity of labor supply. When the total-income elasticity is larger (in absolute value), the demogrant leads to larger reductions in labor supply. As a result, when $\Phi = 2.5$, the tax-rate increase that is needed to finance the demogrant will be greater, and the MECR increases sharply, to 95%.

When $\Phi = 5.0$ for every consumer group, the weighted average of the implied total-income elasticities increases further, to -0.739, which is much larger than virtually any of the static econometric estimates of this parameter. When $\Phi = 5.0$ for every group, the MECR rises to 131.2%. However, it would probably be unwise to take this result very seriously, because it is based on such an enormous value for the total-income elasticity of labor supply.

We have now used two static models to show that the time-endowment parameter can have a substantial effect. In a one-consumer model, the simulated excess burden of a labor tax depends critically on the total-income elasticity of labor supply, which in turn depends on the time endowment. In a multi-consumer model, the simulated marginal efficiency cost of redistribution also depends on the total-income elasticity of labor supply, and therefore on the time endowment. In either case, the large values of the time-endowment parameter that have been used in some simulation models will lead to unacceptably large total-income elasticities of labor supply, and consequently to unrealistically large simulated efficiency costs.

Static models are useful for addressing a number of questions. However, much of the effort of simulation modelers has justifiably been concentrated on dynamic models. We will turn

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[10] Of course, this does not necessarily mean that a program such as this will reduce social welfare. That will depend on the degree of concavity of the social-welfare function. For discussion, see Ballard (1988).

our attention to dynamic models in the next two sections of this paper. At this point, however, it is appropriate to mention Fullerton, Henderson, and Shoven (1984), who use the GEMTAP model, which is described in detail in Ballard, Fullerton, Shoven, and Whalley (1985). This model has a savings decision, which gives the model an important dynamic element. However, the labor-supply decision in the model is static, so that the labor-supply specification of GEMTAP is comparable with the labor-supply specifications of the models discussed so far in this paper.

Fullerton, Henderson, and Shoven consider a policy proposal that would integrate the personal and corporate income taxes in the United States. They use a data set that represents the U.S. economy for 1973. The tax revenue lost through corporate tax integration is recovered by increasing the marginal tax rates in the personal income tax in the model. Since the personal income tax falls largely on labor earnings, the labor-supply elasticities are important for the results. In the standard version of the GEMTAP model, Φ is set to 1.75. In this case, the present discounted value of the welfare gain from the tax-policy change is simulated to be $344.4 billion, in 1973 dollars. However, when the authors reduce Φ to 1.25 (which was used by Piggott and Whalley (1982)), the simulated welfare gain increases to $512.5 billion, which is an increase of nearly 49 percent. This change in the results makes sense in terms of the analysis above: When Φ is smaller, the compensated labor-supply elasticity is smaller. In a simulation experiment such as this one, the adverse welfare effects of increased labor taxes will be smaller when the compensated labor-supply elasticity is smaller. Thus, when Φ is smaller, the loss from replacing tax revenue with higher labor taxes is smaller, and the simulated net gain from corporate tax integration increases.

The calculations reported by Fullerton, Henderson, and Shoven are based on a model with 12 different consumer groups. Since the relationship between Φ and η_I depends on a variety of factors, the GEMTAP modelers' practice of using a single value for Φ for all consumer groups will lead to different values of η_I for different consumer groups. However, as reported in Ballard (1990), the weighted average of the total-income elasticities is approximately –0.33, which is quite large in absolute value.

Based on the earlier analysis, my view is that Piggott and Whalley were closer to the correct value of Φ, with $\Phi = 1.25$. Thus, my suggestion is that the GEMTAP modelers were incorrect when they chose 1.75 as their standard value for Φ, even though some of my own writings employed this value. The purpose of this paper is not to defend my own prior results, nor is it my purpose to point an accusing finger at the results of some other researcher. Instead, the point is that *most* researchers (including myself in much of my early work) have specified the time-endowment parameter arbitrarily. When a researcher specifies Φ on the basis of some arbitrary statement about the number of hours available, without paying attention to the implied elasticities, there is simply no guarantee that sensible values will be chosen.

III. The Effects of the Time Endowment in an Infinite-Horizon Model

One simple way to represent dynamic choices is to assume that the economy contains one or more infinitely lived consumers. Obviously, a researcher using this type of model will not be able to address questions regarding the intergenerational distribution of gains and losses from tax-policy changes. However, an infinite-horizon model is much simpler than an otherwise-comparable overlapping-generations model. For this reason, infinite-horizon models have retained a great deal of popularity. For example, see Greenwood and Huffman (1991), Jorgenson and Wilcoxen (1998), Jorgenson and Yun (1990), Judd (1985), Lucas (1990), Mendoza and Tesar (1998), and many others.

In this section, I will discuss the effects of Φ in an infinite-horizon simulation model based on Ballard and Goulder (1985), who used such a model to assess the efficiency effects of tax-policy proposals that would move the U.S. tax system toward greater reliance on consumption taxation. As in most infinite-horizon models, utility is assumed to be additively separable over time. Ballard and Goulder use the following utility functional:

$$(16)\quad U = \sum_{t=1}^{\infty} \frac{1}{(1+\rho)^{t-1}} \frac{\left[(C_t - C_t^*)^{\alpha} \ \ell_t^{\ 1-\alpha}\right]^{\delta}}{\delta},$$

where δ is one minus the inverse of the intertemporal elasticity of substitution, α is a weighting parameter, and ρ is the rate of time preference. As in the static model described earlier, C represents consumption and ℓ represents leisure. C^*_t is the minimum required level of consumption in period t. During the course of their research, Ballard and Goulder found that saving could be extraordinarily responsive to changes in the rate of return, and they incorporated the C*'s in an attempt to reduce the responses to more realistic levels. In much of what follows, C^*_t will be set to zero, for all *t*. However, I include C^*_t's in the algebraic derivations, so that the reader can see the ease with which these parameters can be incorporated, as well as the effects that they will have on the model.

The consumer is assumed to maximize equation (16), subject to a wealth constraint:

$$(17)\quad \sum_{t=1}^{\infty} \frac{P_t\ C_t}{\overset{t}{\underset{s=1}{\pi}} \left(1+r_s\right)} = K_1 + \sum_{t=1}^{\infty} \frac{W_t H_t + TR_t}{\overset{t}{\underset{s=1}{\pi}} \left(1+r_s\right)}$$

where K_1 is the value of initial capital, TR_t is the value of transfers received in period t, $r_1 \equiv 0$, and r_s is the rate of return in period s, for s>1.

Equations (16) and (17) can be used to form a Lagrangean function. After the Lagrangean has been formed, it is possible to take the first-order conditions with respect to consumption and leisure in any period *t*. Then, we divide the first-order condition for consumption in period *t* by the first-order condition for leisure in period *t*, and solve for leisure in period *t*, to derive an expression that shows the relationship between leisure and discretionary consumption in any period:

$$(18)\quad \ell_t = (C_t - C_t^*)\ ((1-\alpha)/\alpha)\ (P_t/W_t).$$

Substituting equation (18) into the first-order condition for consumption, and dividing the resulting expression for period *t* by the corresponding expression for period 1, we generate the equation that describes the optimal path of discretionary consumption. This relates discretionary consumption in period *t* to discretionary consumption in period 1:

$$(19)\quad (C_t - C_t^*) = (C_1 - C_1^*)\ \Omega_t\ ,$$

where

$$\Omega_t = \Omega_{1t}^{(1/(1-\delta))}\Omega_{2t}^{(\delta(1-\alpha))/(\delta-1)} ,$$

where

$$\Omega_{1t} = \left(\frac{P_1}{P_t}\right)\left(\frac{\overset{t}{\underset{s=1}{\pi}}(1+r_s)}{(1+\rho)^{t-1}}\right)$$

and

$$\Omega_{2t} = \left(\frac{W_t}{P_t}\right)\left(\frac{P_1}{W_1}\right).$$

Equation (19) shows that consumption will grow more rapidly when the difference between the rate of return and the rate of time preference is larger, and that the rate of growth is also influenced by the intertemporal substitution elasticity.

After some further manipulations, we can derive an expression for first-period discretionary consumption in terms of all of the parameters of the problem:

(20) $$C_1 - C_1^* = \frac{\sum_{t=1}^{\infty}\left[\left(W_tE_t + Y_t - P_tC_t^*\right)\overset{t}{\underset{s=1}{\pi}}\frac{1}{(1+r_s)}\right] + K_1}{\frac{1}{\alpha}\sum_{t=1}^{\infty}\left[P_t\,\Omega_t \quad \overset{t}{\underset{s=1}{\pi}}\frac{1}{(1+r_s)}\right]}$$

Equation (20) reveals a very important property of intertemporal models such as this one. The numerator of the right-hand side of the equation contains the present discounted value of the consumer's lifetime resources, less the present discounted value of the consumer's lifetime stream of expenditure on required consumption. In a steady-state infinite-horizon model such as this one, saving will be positive in every period. Therefore, the present value of lifetime resources less required consumption expenditure will be positive. Since every period's consumption is a normal good, and every period's leisure is also a normal good, anything that changes the present discounted value of lifetime resources less required consumption expenditure will have a direct impact on first-period consumption and leisure. If there is a *decrease* in the net rate of return, it would lead to an *increase in* the present value of lifetime resources less required consumption expenditure, and the consumer will want to consume more goods and more leisure in the first period.

However, in the past few decades, far more attention has been focused on policies which would *increase* the net rate of return, such as moves toward greater reliance of consumption taxation, and/or decreased taxation of capital income. When the net rate of return increases, the present value of lifetime resources decreases. As a result, the consumer will want to decrease consumption of goods in the first period. In addition, the consumer will want to decrease first-period leisure, *i.e.*, the consumer will want to work more. Of course, if goods consumption is reduced at the same time that labor supply is increased, saving must increase.

These effects on labor supply and saving can be very large. Starrett (1981) was among the first to appreciate what really happens in this type of intertemporal model. Starrett suggests the introduction of a consumption floor, which introduces an element of complementarity across time which is otherwise totally absent from additively separable models of this type. As Starrett (p. 12) puts it:

> Without complementarity, substitution effects ‘compound’ across time; the rate at which a consumer can trade consumption many periods hence for consumption now is very sensitive to the rate of interest and (in the absence of complementarity) all consumers will engage in a lot of substitution unless [the intertemporal substitution elasticity is extremely small].

Starrett's analysis was directed specifically at the overlapping-generations work of Summers (1981). However, his comments are even more appropriate for the analysis of an infinite-horizon model, because the infinite horizon means that the effects on the present value of lifetime resources can be even larger than they are in an overlapping-generations framework.

The size of these effects will be determined by a number of parameters, including the intertemporal substitution elasticity, the base-case rate of return, the rate of time preference, and the amount of minimum required consumption. However, the time-endowment parameter also plays an important role, for two reasons. First, Φ places an upper bound on the amount by which labor supply can increase. If Φ is 1.1, then the labor supply in any period of the revised-case scenario cannot be more than 10% larger than the labor supply in the corresponding period of the base-case scenario. Second, Φ has an important effect on the present discounted value of lifetime resources.

Table 3 shows that Φ can have an important effect on the results. The results in Table 3 are generated from an updated version of the infinite-horizon model of Ballard and Goulder (1985), using data for the United States for 1983. See Scholz (1987) for a description of the data and their sources. The structure of production in this model is the same as the structure of production in Ballard, Fullerton, Shoven, and Whalley (1985), although the structure of consumer choice is substantially different. The policy investigated here involves a move toward greater reliance on consumption taxation, in which tax-deferred status is extended to all financial saving. For more details, see Ballard, Fullerton, Shoven, and Whalley (1985, especially Chapter 9).

We can use the model to calculate the implied elasticity of labor supply with respect to a permanent increase in the net rate of return. Since the extent of the response differs from period to period, I report the elasticity of *first-period* labor supply with respect to the net rate of return. The results are shown in the middle column of Table 3. Even for a very low value of Φ, such as 1.05, there is a positive response of labor supply. However, the implied elasticity of about 0.08 is not exceptionally large. However, as Φ increases, so does the elasticity. When Φ is 2.5, the elasticity of first-period labor supply with respect to a permanent increase in the net rate of return is more than 2.5. When Φ is 5.0, the elasticity increases to more than 6.9. Because this response is so exceptionally large, it is reasonable to question whether these results are useful.

In the third column of Table 3, I present the welfare gain from adopting a consumption tax, as a percentage of GDP in the base case. Not surprisingly, when the model generates more elastic responses, the welfare changes are larger as well.[11]

IV. The Effects of the Time Endowment in an Overlapping-Generations Life-Cycle Model

Infinite-horizon models, such as the one discussed in the previous section, can help to provide an understanding of intertemporal issues. However, these models face some obvious and very serious handicaps. Death does not visit the consumers in an infinite-horizon model, but

[11] As noted before, Jorgenson and Wilcoxen (1998) use an average value of ø=4.1. This may contribute to the very large labor-supply responses generated by their model. They find that, if the United States were to adopt a pure consumption tax, first-period labor supply would increase by 30 percent.

real people die. Thus, a real person's planning horizon may be very different from the planning horizon implied by an infinite-horizon model. Moreover, the infinite-horizon model is completely incapable of assessing issues of intergenerational equity and efficiency. Consequently, overlapping-generations life-cycle models are an attractive vehicle for analyzing dynamic tax-policy questions.

In this section, as in previous sections, I will discuss models on which I have worked in the past. My interest in overlapping-generations models began with my dissertation (Ballard (1983)), and continued with Ballard and Goulder (1987) and Ballard and Kim (1995, 1996). Here, I will focus specifically on the version of the model developed in Ballard and Kim.

It is appropriate to compare the Ballard-Kim model with the well-known series of overlapping-generations models developed by Auerbach and Kotlikoff (1983, 1987), and their colleagues (such as Auerbach, Kotlikoff, and Skinner (1983), and Altig *et al.* (1997)). There are a large number of similarities between the most recent models of Auerbach, Kotlikoff, and their colleagues, and the Ballard-Kim model. In both cases, the consumers maximize a lifetime utility functional that depends on consumption in each period, leisure in each period, and bequests. One difference is the fact that the Ballard-Kim model uses a multi-sector data set, whereas the model of Auerbach, *et al.*, has only a single sector. However, in many cases, the degree of sectoral disaggregation does not have a large effect on the results. In particular, the focus of this paper is on the effects of the time-endowment parameter, which are not altered significantly by the degree of sectoral disaggregation.

Another difference is that Auerbach, *et al.*, calculate dynamic sequences of equilibria with perfect foresight, whereas the current version of the Ballard-Kim model calculates sequences of equilibria with myopic expectations. Ballard and Goulder (1985) and Ballard

(1987) discuss the effects of price expectations, in an infinite-horizon model and in the GEMTAP model. They show that the differences between perfect foresight and myopia are not very large, for certain simulation experiments. On the other hand, Auerbach and Kotlikoff (1987) and Judd (1985) show that the nature of expectations may be important if an announcement effect is involved, or if the policy change is only temporary. However, in this paper, we concentrate exclusively on permanent, unanticipated policy changes, so that announcement effects are not relevant. In future work, I intend to incorporate a perfect-foresight algorithm.

We assume that, in every period, aggregate consumption, saving, and labor supply are derived from the intertemporal optimizing behavior of individual generations. Each generation or cohort has an economic life of 55 years (for example, from age 21 through age 75), and a new cohort is "born" each period (one period is five years).[12] Thus, in any period, household decisions are being made by 11 cohorts of different ages.

Households derive utility from consumption, leisure, and the giving of bequests.[13] The utility function for any given cohort takes the following form:

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[12] Auerbach and Kotlikoff also assume an economic lifetime of 55 years. However, they calculate equilibria every year. The assumption of a five-year period reduces computational expense, without sacrificing a great deal of information.

[13] White (1978), Mirer (1979), and Kotlikoff and Summers (1981) all found that the simple life-cycle model (with no bequests) does not accord with the facts. For example, elderly consumers tend to dissave much more slowly than would be predicted by the simple life-cycle model. Gale and Scholz (1994) conclude that *inter vivos* transfers and bequests may account for about 51 percent of net worth accumulation. This implies that an overlapping-generations model will have great difficulty in capturing the stylized facts of the economy, unless it incorporates bequests.

(21) $$U=\sum_{t=1}^{T}\frac{1}{(1+\rho)^{t-1}}\left\{\frac{\left(C_t-C^*\right)^{\sigma}+\alpha_t\,\ell_t^{\ \sigma}}{\delta}\right\}^{\frac{\delta}{\sigma}}+\frac{1}{(1+\rho)^{T}}\frac{b^{1-\delta}B^{\delta}}{\delta}.$$

In the above expression, *t* is the period, and *T* is the index for the last period of life. As in the infinite-horizon model, C_t is consumption in period *t*, C^* is minimum required consumption, ℓ_t is leisure in period *t*, and ρ is the rate of time preference. The parameter $\sigma \equiv 1 - 1/\overline{\sigma}$, where $\overline{\sigma}$ is the elasticity of substitution between *C* and *l* in a given period. The parameter $\delta \equiv 1 - 1/\overline{\delta}$, where $\overline{\delta}$ is the elasticity of substitution between bundles of *C* and *l* across periods. To maintain dynamic consistency, the elasticity of substitution between consumption/leisure bundles and bequests is also $\overline{\delta}$. The distribution parameter α influences the intensity of demand for leisure at given relative prices, and is related to the time-endowment parameter.

B is the bequest left at the end of year *T*.[14] The parameter *b* determines the strength of the bequest motive. When *b* is zero, individuals derive no benefits from the giving of bequests. Thus, since length of life is assumed to be known with certainty, such that accidental bequests are ruled out, the consumers will not leave any bequests when *b*=0. This type of bequest model has been used, for example, by Blinder (1974). However, it should be noted that other plausible explanations for bequests have been proposed.[15]

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[14] We assume certain date of death, as did Auerbach and Kotlikoff. We also assume that all bequests come at end of life. We abstract from gifts *inter-vivos.* However, it should be noted that Gale and Scholz (1994) estimate *inter-vivos transfers* account for at least 20% of net worth.

[15] Davies (1981) suggests that consumers do not gain utility from bequests, but rather that they are forced to leave accidental bequests as a result of the lack of well-functioning annuities markets. Bernheim, Shleifer, and Summers (1985) suggest that bequests are a device by which parents manipulate the behavior of their children. Barro (1974) regards bequests as arising from

Each cohort maximizes utility subject to an intertemporal wealth constraint. Suppressing taxes for expositional convenience, we can write the lifetime wealth constraint as:

$$(22) \quad \sum_{t=1}^{T} \{P_t C_t\} d_t + P_B B d_T = K_1 + \sum_{t=1}^{T} \{W_t H_t + TR_t + IN_t\} d_t \,,$$

where K_1 is the value of the initial capital endowment, W_t is the net wage in period t, TR_t is transfers in period t, and IN_t represents inheritances in period t. The variable P_t refers to the price index for consumption, which is a weighted average of the prices of specific consumption goods purchased in the given period. P_B is the "price of bequests," which is the price of a unit of capital at the end of the consumer's life. The discounting operator for period t, d_t, is defined by

$$d_t \equiv \begin{cases} \dfrac{1}{\prod_{s=1}^{t-1}(1+r_s)} & , \; \forall\, t > 1 \\[2ex] 1 & , \; t = 1 \end{cases} ,$$

where r_s is the expected rate of return between period s and period $(s+1)$. Equation (22) thus states that the sum of current non-human wealth and the present value of prospective lifetime labor income, transfers, and inheritances must equal the present value of consumption plus bequests. This wealth constraint for the OLG model is very similar to the wealth constraint for the infinite-horizon model (equation (17)). The only differences are the length of the time horizon and the presence of bequests and inheritances in equation (22).

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the decisions of intergenerationally altruistic individuals. Such individuals maximize a utility stream which includes the utilities of their immediate descendants as well as themselves.

If no constraint were imposed on the consumers in the model, it is entirely possible that they would desire to provide negative amounts of labor. Consequently, we impose the constraint that labor supply must be non-negative in every period:

(23) $$H_t \geq 0, \text{ for all } t.$$

Each cohort has a given endowment of potential labor time, E, which is allocated to working and leisure: $E = H_t + \ell_t$. The value of E is constant over the lifetime of a given cohort. The hourly wage (W_t) can be written as

(24) $$W_t = W_t' e_h \ ,$$

where W_t' is the prevailing wage per unit of effective labor, and e_h is the ratio of effective labor to labor hours for a cohort of age *h*. The labor-efficiency ratio (e_h) changes over the lifetime of a given cohort, reflecting the fact that the skill level of workers will change as they age.

The consumer's choice variables include consumption in each period (C_t) and leisure in each period $(\ell_t, \text{or } E - H_t)$, which were also choice variables in the infinite-horizon model. Another choice variable in the overlapping-generations model is the size of the bequest (*B*), which did not enter the infinite-horizon model. We can form the Lagrangean function by combining equations (21), (22), and (23):

$$\textbf{(25)}\ \ L = \sum_{t=1}^{T} \frac{1}{(1+\rho)^{t-1}} \frac{\left\{\left(C_t - C^*\right)^{\sigma} + \alpha_t \ell_t^{\ \sigma}\right\}^{\frac{\delta}{\sigma}}}{\delta} + \frac{1}{(1+\rho)^{T}} \frac{b^{1-\delta} B^{\delta}}{\delta}$$

$$+ \lambda\left[K_1 + \sum_{t=1}^{T}\left\{W_t\left(E - \ell_t\right) + TR_t + IN_t - P_t C_t\right\} d_t - P_B\ B\ d_T\right]$$

$$+ \lambda\left\{\sum_{t=1}^{T} \mu_t\ H_t d_t\right\}\ ,$$

where λ is a Lagrange multiplier that represents the marginal utility of lifetime resources, and the μ_t's are the Kuhn-Tucker multipliers on the constraints on labor supply.

We define $\hat{C}_t$ = ($C_t - C_t$*). In other words, $\hat{C}_t$ is discretionary consumption in period *t*. Taking the first-order conditions for consumption and leisure, and rearranging, gives us the following expressions:

$$(26) \qquad \frac{1}{(1+\rho)^{t-1}}\left(\hat{C}_t^{\sigma} + \alpha_t \ell_t^{\sigma}\right)^{\frac{\delta}{\sigma}-1} \hat{C}_t^{\sigma-1} = \lambda P_t d_t \quad ,$$

and

$$(27) \qquad \frac{1}{(1+\rho)^{t-1}}\left(\hat{C}_t^{\sigma} + \alpha_t \ell_t^{\sigma}\right)^{\frac{\delta}{\sigma}-1} \alpha_t \ell_t^{\sigma} = \lambda W_t d_t + \lambda \mu_t d_t \ .$$

When $\mu_t = 0$, we have positive labor supply, and thus, W_t is the effective wage. If we have zero labor supply, then $\mu_t > 0$, and $(W_t + \mu_t)$ is the reservation wage at which the consumer would choose to supply exactly zero labor. Equation (26) indicates that the marginal utility of consumption at time *t* must equal the marginal cost of consumption at time *t*, and equation (27) shows that the marginal utility of the leisure at time *t* must equal its marginal opportunity cost.

Dividing (26) by (27) and arranging, we solve for ℓ_t (leisure in period *t*), as a function of consumption in period *t* and various parameters:

(28) $$\ell_t = \hat{C}_t \xi_t \ ,$$

where

$$\xi_t = \left(\frac{W_t + \mu_t}{\alpha_t P_t} \right)^{\frac{1}{\sigma - 1}} .$$

Equation (28) serves the same purpose for the OLG model that was served for the infinite-horizon model by equation (18). The two equations differ because the two models use slightly different functional forms, and because the OLG model incorporates an explicit non-negativity constraint on labor supply[16]. Substituting (28) into (26) and manipulating terms gives us:

[16] An interesting subject for future work will be to make the within-period utility functions of the infinite-horizon model and the OLG model the same. This would simplify the task of isolating the effects of the length of the time horizon.

(29) $$\hat{C}_t = \lambda^{\frac{1}{\delta-1}} P_t^{\frac{1}{\delta-1}} \left(\frac{\prod_{s=1}^{t-1}(1+r_s)}{(1+\rho)^{t-1}} \right)^{\frac{1}{1-\delta}} \left(1+\alpha_t \xi_t^{\sigma}\right)^{\left(1-\frac{\delta}{\sigma}\right)\left(\frac{1}{\delta-1}\right)} .$$

Dividing (29) for period *t* by (29) for period (*t*-1), we have

(30) $$\frac{\hat{C}_t}{\hat{C}_{t-1}} = (1+g_t)\left(\frac{P_t}{P_{t-1}}\right)^{\frac{1}{\delta-1}} \left(\frac{1+\alpha_t \xi_t^{\sigma}}{1+\alpha_{t-1}\xi_{t-1}^{\sigma}}\right)^{\psi} ,$$

where

$$\begin{cases} g_t = \left(\frac{1+r_{t-1}}{1+\rho}\right)^{\frac{1}{1-\delta}} -1, \ \textit{i.e.}\text{, the reference growth rate of consumption,} \\ \text{and} \\ \psi = \left(1-\frac{\delta}{\sigma}\right)\left(\frac{1}{\delta-1}\right) = \left(\frac{\sigma-\delta}{\sigma}\right)\left(\frac{1}{\delta-1}\right) = \frac{\sigma-\delta}{\sigma(\delta-1)} . \end{cases}$$

By recursively applying (30) over successive periods and manipulating, we can express $\hat{C}_t$ in terms of $\hat{C}_1$ and the parameters of the problem:

(31) $$\hat{C}_t = \hat{C}_1 \Omega_t \quad ,$$

where

$$\Omega_t = \left\{ \left(\frac{P_t}{P_1} \right) (1+\rho)^{t-1} d_t \right\}^{\frac{1}{\delta - 1}} \left(\frac{1 + \alpha_t \xi_t^{\sigma}}{1 + \alpha_1 \xi_1^{\sigma}} \right)^{\psi} .$$

Equation (31) represents an optimal consumption path. Once the optimal $\hat{C}_1$ is known, we can obtain an optimal consumption path conditional on expected prices and interest rates. It can be seen that equation (31) is very similar to equation (19), the optimal consumption path in the infinite-horizon model. The differences between equations (19) and (31) are the result of slight differences in functional form, as well as the non-negativity constraint on labor supply.

Differentiating the Lagrangean function with respect to bequests (*B*) yields:

(32) $$\frac{1}{(1+\rho)^T} b^{1-\delta} B^{\delta - 1} = \lambda P_B d_T \quad ,$$

which indicates that the marginal utility of the bequest must equal its marginal opportunity cost. Rearranging equation (26) gives us:

$$(33) \qquad \lambda = \frac{1}{P_T d_T} \frac{1}{(1+\rho)^{T-1}} \left(\hat{C}_T^{\sigma} + \alpha_T \ell_T^{\sigma}\right)^{\frac{\delta}{\sigma}-1} \hat{C}_T^{\sigma-1} .$$

Substituting (28) and (31) into (33) and rearranging terms yields the following expression:

$$(34) \qquad \lambda = \left((1+\rho)^{T-1} P_T d_T\right)^{-1} \left(1+\alpha_T \xi_T^{\sigma}\right)^{\frac{\delta}{\sigma}-1} \left(\hat{C}_1 \Omega_T\right)^{\delta-1} .$$

Substituting (34) into (32) and rearranging terms gives us an expression for the optimal bequest in terms of discretionary consumption in the base period:

$$(35) \qquad B = b\omega \hat{C}_1 \Omega_T ,$$

where

$$\omega = \left(\frac{(1+\rho)P_B}{P_T}\right)^{\frac{1}{\delta-1}} \left(1+\alpha_T \xi_T^{\sigma}\right)^{\frac{\delta-\sigma}{\sigma(\delta-1)}} .$$

Equation (35) implies that bequests are equal to zero when the bequest intensity parameter (b) is zero, and that bequests increase with b. Although equation (35) appears to suggest a linear relationship between bequests and b, the relationship is actually non-linear, since higher values of b imply lower levels of discretionary consumption. (The consumer must reduce consumption in order to leave a larger bequest.)

Substituting (31) into (28), we have

(36) $$\ell_t = \hat{C}_1 \Omega_t \xi_t \;.$$

From equation (31), we have

(37) $$C_t = C^* + \left(C_1 - C^*\right)\Omega_t \;.$$

Substituting (31), (35), (36), and (37) into (22), and rearranging terms, gives us an initial optimal consumption:

(38) $$C_1 = \mathrm{C}^* + \frac{K_1 + \sum_{t=1}^{T}\left\{\left(W_t + \mu_t\right)E + TR_t + IN_t - P_t C^*\right\}d_t}{\sum_{t=1}^{T}\Omega_t\left\{\left(W_t + \mu_t\right)\xi_t + P_t\right\}d_t + P_B b\omega\,\Omega_T d_T} \;.$$

In equation (38), first-period consumption $\left(C_1\right)$ is linearly homogeneous in lifetime resources (initial wealth plus the present value of lifetime potential labor time, transfers, and inheritances). Equations (38) and (30) imply that, for given lifetime resources and prices, a lower *b* indicates higher consumption at each point in time.

Once we get the initial consumption level ($\hat{C}_1$), we can calculate an optimal consumption path according to (31). By substituting this optimal consumption path into (28) and the leisure constraint, we can get the optimal leisure path and thus the optimal labor path:

$$\text{(39)} \qquad \left[\begin{array}{l} \mathrm{C_t} = \mathrm{C}^* + \left(\mathrm{C_1} - \mathrm{C}^*\right)\Omega_{\mathrm{t}} \\ \ell_{\mathrm{t}} = \left(\mathrm{C_t} - \mathrm{C}^*\right)\xi_{\mathrm{t}} \\ \mathrm{H_t} = \mathrm{E} - \ell_{\mathrm{t}} \end{array}\right.$$

From equation (30), the rate of consumption growth is negatively related to the growth rate of prices (P_t / P_{t-1}) and positively related to the interest rate (r_t) and to the growth rate in the real wage. In the steady-state, $P_t = \overline{P}$ and $r_t = \overline{r}$, and the consumption growth equation becomes:

$$\text{(40)} \qquad \frac{\hat{C}_t}{\hat{C}_{t-1}} = \left(1+g\right) \left[\frac{1+\alpha_t \xi_t^{\sigma}}{1+\alpha_{t-1}\xi_{t-1}^{\sigma}}\right]^{\psi},$$

where the steady-state reference growth rate of consumption (g) is:

(41) $$g = \left(\frac{1+\bar{r}}{1+\rho} \right)^{\frac{1}{1-\delta}} - 1.$$

Thus, lower values for the rate of time preference (ρ) or higher values for the intertemporal elasticity of substitution ($\bar{\delta}$), which is inversely related to δ, imply a steeper consumption profile in the steady state. Although the growth rate of aggregate consumption is a constant in the steady state, the growth rate of individual consumption is not. Individual consumption growth will depend positively on the hourly wage, W_t (or $W'_t\, e_h$), and this in turn will vary over one's lifetime according to changes in e_h. These variations imply that the bracketed component of equation (40) will not be constant over time. Thus the growth rate of individual consumption changes over the lifetime.

From (28), leisure is related to discretionary consumption according to:

(42) $$\ell_t = \left(C_t - C^* \right) \left(\frac{W'_t + \mu_t}{\alpha_t P_t} \right)^{\frac{1}{\sigma - 1}}.$$

Thus, with σ<1, leisure in period *t* is positively related to P_t and negatively related to W_t. The optimal labor path (H_t) is negatively related to P_t and positively related to W_t, since $H_t = E - \ell_t$.

In the base case, the model generates a steady-state growth path. Relative prices remain constant through time, and all outputs and aggregate incomes grow at the growth rate of the overall economy. In addition, government spending, tax revenue, government deficits, and government debt all remain in constant proportion to GNP. In the revised case, we maintain the same growth of government debt as in the base case, but examine the effects of changing the configuration of tax rates, while holding the profiles of real government spending and bond issue fixed.[17] The profiles of both exhaustive government expenditures and transfer payments are held constant in real terms during revised-case simulations.

As with the infinite-horizon model described earlier, the basic source of data for the OLG model is Scholz (1987). Scholz's original data set provides information for 14 consumer groups, which are distinguished on the basis of their incomes in 1983. We begin by aggregating them into a single consumer group. Then, we re-divide them into 11 groups, which are distinguished by age. For inheritances and government transfers, we make the following adjustments. The bequest left by one cohort is assumed to be divided among the 11 cohorts that are alive during the next period. Therefore, we need to calculate the proportions of total bequests that are received by the next 11 cohorts. We derive these proportions based on Consumer Expenditure Survey data which Projector and Weiss (1966) used in their unpublished work sheets. We also need the proportions of government transfers that are received by the different cohorts at a given time. These proportions are derived from Michigan Panel Studies of Income Dynamics Survey data (PSID, 1973). These data were collated by Charles Becker, and reported in Fullerton, Shoven, and Whalley (1980). However, since these data on inheritances and transfers are for age groups ten years apart, we average the data so that they can conform to the age brackets of our model. Table 4 presents the proportions we use for inheritances and transfers.

Another important input into this type of OLG model is the path of effective wage rates over the life cycle. The "human-capital earnings function" of Mincer (1970, 1974), in which

[17]The model of bonds and debt used in this study is essentially the same as that described in a paper by Goulder (1985), which allows for alternative government financing and contains the overlapping-generations features described in this paper. With the exception of the treatment of bond issuance and debt, the modeling of government is similar to that in Ballard, Fullerton, Shoven, and Whalley (1985), and in Ballard and Goulder (1987).

earnings are expressed as a quadratic in potential experience, has been one of the most widely accepted empirical specifications in economics. Mincer expresses the natural logarithm of earnings per hour, week, or year as a linear function of the number of years of school completed and as a quadratic function of years since leaving school (or potential work experience).

Murphy and Welch (1990) find that, in spite of its widespread acceptance, the quadratic human-capital earnings function provides a poor approximation of the true empirical relationship between earnings and experience. They note that the standard quadratic function in experience understates early career earnings growth by about 30% - 50%, and overstates mid-career growth by 20% - 50%. They suggest that a quartic specification provides a reasonably good approximation to the "true" earnings function. The model used here incorporates exogenous values for e_h, based on a quartic function estimated by Murphy and Welch:

$$e_h = e^x \quad , \tag{43}$$

where $$\mathrm{x} = \mathrm{b}_0 + \mathrm{b}_1\mathrm{h} + \mathrm{b}_2\mathrm{h}^2 + \mathrm{b}_3\mathrm{h}^3 + \mathrm{b}_4\mathrm{h}^4 \quad .$$

We use b_0 = -0.465218, b_1 = 0.584478, b_2 = -0.1319, b_3 = 0.014343, and b_4 = -0.000624.

Now that we have described the model, we turn to the calibration methods. One important part of the calibration procedures used by Ballard and Kim for their OLG model is the replication requirement, which is familiar from other simulation models. In the base case, the integrated model is required to generate an equilibrium solution that replicates the 1983 data set. In particular, the aggregate labor supply, wage income, capital income, consumption, and saving that emerge from the overlapping-generations model must equal the aggregate labor demand, wage payments, capital income payments, consumption, and saving contained in the 1983 GEMTAP data set. In calibrating their OLG model, Ballard and Kim also impose a balanced-growth requirement: In the base case, the model is constrained to generate a steady-state growth path.

The elasticities of substitution, $\overline{\sigma}$ and $\overline{\delta}$, are set exogenously. Summers (1982) favors a value for $\overline{\delta}$ of around 0.33, and Ghez and Becker (1975) estimate that $\overline{\delta}$ should be at most 0.28. Hall (1988) finds that $\overline{\delta}$ is indistinguishable from 0. However, Hansen and Singleton (1983) and Weber (1970, 1975) call for values equal to or greater than 0.5 for $\overline{\delta}$.

It is important to recognize, however, that the choice of $\overline{\delta}$ (the intertemporal elasticity of substitution between and leisure) cannot be made independently of the choices of other parameters, especially $\overline{\sigma}$ (the elasticity of substitution of substitution between consumption and leisure in any period) and b (the bequest parameter). Moreover, even for seemingly reasonable values for $\overline{\delta}$, models of this type can generate unrealistically large saving elasticities. This is because of "human-wealth effect," which was discussed above in the section on the infinite-horizon model. The human-wealth effect may be less severe in an OLG model than in an infinite-horizon model, but it can still be very powerful. From equation (28), we know that, when consumption decreases, leisure decreases as well. Thus, not only does the consumer buy fewer goods, but she also works more and earns more labor income. These two effects can generate very large saving elasticities. In our simulation model, we want to use values for $\overline{\sigma}$ and $\overline{\delta}$ that are reasonably in line with those from the econometric literature. But we are also mindful of the need for implied elasticities of behavioral response to be reasonable. Equation (41) indicates that the steady-state reference growth rate of consumption (g) is determined by δ, ρ, and $\bar{r}$. We have chosen $\bar{\delta}$=0.4, $\bar{\sigma}$=0.8, and ρ=0.01 (for one year) for the central case. These exogenous parameters imply a value of approximately 0.0129 for g for one year, and 0.0664 for five years.

Given these parameters values, which are taken exogenously from outside econometric estimates, it is necessary to define the leisure-intensity parameter (α_t) and the time endowment and capital endowment for each of the 11 cohorts alive in the benchmark year. Under the calibration procedures chosen by Ballard and Kim, this is done in a way that assures that both the replication and balanced-growth requirements are satisfied. The problem is made tractable by the assumption of balanced growth and the fact that the utility-function parameters of all cohorts are specified to be the same. Under these circumstances, in the steady state, the pattern across time of a given cohort's consumption (or leisure) will be closely linked to the pattern across cohorts of consumption (or leisure) at a given point in time. In other words, in a steady state, we can infer information about individual profiles on the basis of cross-section information. In the case of consumption, the following relationship must hold in the steady state:

(44) $$C_{n-t,1} = \frac{C_{n,t+1}}{(1 + g_{pop})^t} .$$

Thus, the total consumption of cohort n-t (born t years before cohort n) in period 1 $(C_{n-t,1})$ must be the same as the total consumption of cohort n in period t+1 $(C_{n,t+1})$, adjusted for the growth rate of population (g_{pop}). Thus, for purposes of calibration, equation (42) allows the expression of the behavior of all cohorts in terms of the behavior of a single representative cohort.

Once the consumption of the representative cohort is determined, using equation (44), it would appear that we should be able to determine the leisure of the representative cohort immediately, by using equation (28). However, the leisure-intensity parameter, α_t, is on the right-hand side of equation (28). Therefore, the consumption level of the representative cohort actually depends on the value of α_t. This means that we have a simultaneity: The time endowment depends on the consumption of the representative cohort, but the consumption of the representative cohort depends on the time endowment. Ballard and Kim use an iterative procedure (described in the appendix of Ballard and Goulder (1987)) to find the combination of consumption level and leisure-intensity parameter that satisfies both relationships simultaneously.

Based on the earlier discussions in this paper, the problem with this procedure is obvious. Instead of specifying the time-endowment parameter exogenously, based on concern for the responsiveness of labor supply in the model, this procedure specifies the time-endowment parameter in a way that may potentially lead to very large values.

In fact, this is unfortunately what has happened. Of course, the exact values of Φ that are used will depend on the other parameters, but the Ballard-Kim model often generates values of Φ that are in excess of 4.0. Not surprisingly, this very large time-endowment parameter can lead to some startling results.

Ballard and Kim simulate the effects of replacing a portion of the income tax in the United States with a value-added tax. Since the interest rate goes up when we adopt a VAT, the present value of human wealth decreases. Then, consumers in the model desire to consume less, and enjoy less leisure. Consequently, labor supply increases. With such large values for Φ, the increase can be extremely large. For example, when C^*=0 and the model is parameterized such that 60 percent of the capital stock is explained by bequests, the price of capital (relative to the price of labor) increases by a whopping 17.3 percent in the first period after the institution of the new policy. The reason for the increase in the price of capital services is that labor has become much more abundant: Labor supply increases by more than 20 percent, and this pushes up the relative price of capital services in the first period. This increase in the price of capital can actually lead to welfare gains for the elderly cohorts.

These results seem highly questionable. I have criticized Auerbach and his colleagues for choosing a value of Φ that leads to excessively large swings of labor supply. However, the calibration procedures of Ballard and Kim are just as problematic. It appears that the problem stems from an overly zealous attempt to replicate the benchmark data. In order to replicate every aspect of the data precisely in an OLG model, the researcher may have to pay a heavy price in terms of other aspects of the model, such as the time endowment, with its subsequent effects on labor-supply elasticities. Auerbach, Kotlikoff, and their colleagues have shown a willingness to settle for what might be called "approximate replication." Given the great complexity of an OLG model of this type, this is probably the best way to go. The next item on my research agenda is to change the calibration procedures, so that a lower value can be chosen for the time-endowment parameter, which will probably make it necessary to abandon some aspect of the replication requirement.

In this paper, I have emphasized that the time-endowment parameter can have an important effect on the results, in static models and in dynamic models. In a static model, when appropriate choices of the time-endowment parameter are combined with standard techniques, it is possible to nail down the compensated and uncompensated labor-supply elasticities with great precision. Thus, if the researcher gives appropriate attention to the time-endowment parameter, all of the important problems of calibration can be solved. However, a great deal more is happening in a dynamic model than in a static model. In a dynamic context, even if Φ is set to a reasonably low value, the intertemporal responses still may be fairly large, especially in the short run. Thus, it will also be important to consider other methods of controlling the behavioral responses in these models. One of these other methods is to introduce adjustment costs in the investment process, which has been done in several of the papers by Auerbach and his co-authors, and by Jorgenson and his co-authors. Another method of controlling the responses is to bring uncertainty into the model, as in Engen and Gale (1993). Thus, the time-endowment parameter is not the *only* important consideration of the calibration process, but it is certainly one important element.

V. Conclusion

In this paper, I have discussed the effects of the time-endowment parameter in four different tax-policy simulation models. In a one-consumer static model, the time-endowment parameter determines the total-income elasticity of labor supply. Therefore, all else equal, the time-endowment parameter controls the compensated labor-supply elasticity. This can have an important effect on the simulated welfare effects of tax-policy changes.

In a multi-consumer static model, the time-endowment parameter once again determines the total-income elasticity of labor supply. If we simulate an increase in transfer payments, this elasticity will determine the amount by which the transfer recipients reduce their labor supply. This, in turn, will have an important effect on calculations of the efficiency costs of redistribution.

In a dynamic model (either an infinite-horizon model or an OLG model), when a tax-policy change leads to an increase in the rate of return, the present value of lifetime resources will decline. As a result, the consumer will reduce consumption. In a model with a labor-supply decision, the consumer will not only reduce consumption of goods, but will also reduce consumption of leisure. In other words, the consumer will work more. The time-endowment parameter plays an important role in determining the size of this effect.

Until now, the most common practice among simulation modelers has been to choose the time-endowment parameter in a fairly arbitrary way, based on *ad hoc* judgments about the number of hours available. In this paper, I have argued that this practice should give way to a new emphasis on choosing the time-endowment parameter for the purpose of controlling the behavioral elasticities implied by the model. If this new procedure is widely adopted in the profession, it will pay dividends in terms of improving the realism and usefulness of simulation models.

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Table 1
Sensitivity Analysis of Results from Ballard (1990)
With Respect to the Time-Endowment Parameter*

Φ	Implied Value of Total-Income Elasticity, η_I	Average Excess Burden**
1.213	-0.1	12.9%
2.5	-0.4421	29.7%
5.0	-0.6787	38.7%

* Other parameters include:

Initial labor tax rate = 0.4
Uncompensated labor-supply elasticity (η) = 0.1

** Average Excess Burden = $(\Delta W / \Delta R) - 1$,

where ΔW = absolute value of the change in consumer welfare from replacing a labor tax with a lump-sum tax of equal revenue yield, and

ΔR = amount of revenue replaced.

Table 2
Sensitivity Analysis of the Results from Ballard and Goddeeris (1996)
With Respect to the Time-Endowment Parameter*

Φ	Implied Value of Total-Income Elasticity, η_I	Marginal Efficiency Cost of Redistribution**
E.g., 1.4	-0.2	41.5%
2.5	-0.5170***	95.0%
5.0	-0.7387***	131.2%

* Other parameters include:

Initial labor tax rate = 0.4
Uncompensated labor-supply elasticities = -0.05 for men
0.20 for women
0.10 for married couples

Weighted average = 0.04043

** Marginal Efficiency Cost of Redistribution is the sum of the losses to those who lose, divided by the sum of the gains for those who benefit, minus one. The policy investigated is a tax-financed demogrant.

*** Weighted Average.

Table 3
Sensitivity Analysis of the Results of an Infinite-Horizon Simulation Model With Respect to Time-Endowment Parameter*

Φ	Elasticity of First-Period Labor Supply With Respect to a Permanent Increase In the Net Rate of Return	Welfare Gain From Adopting a Consumption Tax, As Percent of Base-Case GNP
1.05	0.081	1.48%
1.2	0.326	1.63%
2.5	2.543	2.10%

Table 4

Percentages of Inheritances and Transfers Received by Cohorts of Different Ages

AGE[a]	INPROP[b]	TRPROP[c]
1	4.6%	1.6%
2	6.1	3.2
3	7.6	4.6
4	8.0	6.5
5	8.5	8.3
6	7.4	7.6
7	6.3	7.0
8	9.1	7.0
9	12.1	7.0
10	14.1	23.6
11	16.2	23.6
TOTAL	100.0	100.0

[a]AGE = Period of life. Each period is five years in length, so that the 11 periods of life cover a total of 55 years. When AGE=1, the cohort has just entered the model, and the cohort's last period of life occurs when AGE=11.

[b]INPROP = the proportion of the total bequest that is received by each of the 11 living cohorts in the benchmark.

[c]TRPROP = the proportion of total government transfers that is received by each of the 11 living cohorts in the benchmark.