

# Learning in Legislative Bargaining

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## **Abstract**

The paper considers a legislative bargaining model between an agenda setter (the president) and a legislature (congress). In each period, the president makes a policy proposal, then congress either approves or opposes it. In case of opposition, the president can attempt to change congress' position with some probability that is a function of the president's unobserved ability. Both congress and the president learn about the president's ability over time, which generates the following distortions in the bargaining process: First, it may be optimal for political players to oppose policies that they like. Second, congress may be unwilling to challenge a president, which could lead the president to promote more extreme policies. We then show that these distortions can be minimized if "tough" policy issues are first on the agenda, followed by "easy" policy issues.

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# 1 Introduction

The impact of agenda setters, such as a president, a prime minister, or a party leader in the process of political decision making has been the focus of research at least since the seminal contribution of Romer and Rosenthal (1979). In their paper, the power of the agenda setter is derived from a first-mover advantage. In this paper we investigate another source of power for an agenda setter, the ability to put pressure, cajole and lobby legislators to support a policy proposal. However, agenda setters may differ in their abilities to convince legislators to vote for their proposal, and both legislators and the agenda setter may only learn about the agenda setter's skills in the process of contentious legislative battles.

The skill to convince wavering legislators is undoubtedly an important characteristic of a successful leader. For example, president Lincoln's ability to get the 13th amendment passed in the House, was due to his skills and active involvement in getting the necessary votes. More recently, President Clinton was able to establish good relationships with Republican legislators that helped him to get some of his agenda passed even after the Gingrich revolution. On the other hand, President Obama was often criticized for his frosty relationships with congress, which was at least in part responsible for the lack of passage of an immigration bill.

Our model consists of an agenda setter, referred to as the president, and a legislative body, referred to as congress. The president proposes a policy to congress. Congress members chooses whether or not to support the policy. If there is support then the policy passes immediately. Otherwise, the president can attempt to convince congress members to change their position, but the probability that this attempt succeeds depends on the president's ability. If no policy is passed then all individuals receive a reservation utility. Initially, both congress and the president have the same common prior about the president's ability. In other words, the president learns on the job how to get his agenda passed, and both the president and the congress learn from legislative successes or failures about the president's innate abilities.

As a benchmark we start with the case where only one policy is on the agenda, and hence there is no benefit from learning about the president's ability. We say that a policy issue is not contentious (or informally an easy issue), if the president's most preferred policy choice gives the median legislator a payoff that strictly exceeds the reservation utility. Non-contentious policies pass without any need for the president to intervene. If a policy issue is contentious (i.e., a tough issue), then the president can either moderate the proposal, to make it acceptable to the median legislator, or he can choose not to moderate and instead put pressure on legislators to support the proposal anyway. The first choice is optimal if the president's expected ability is low, whereas the second will occur if the president's ability is expected to be high.

We then assume that there are two policy issues on the agenda. Now learning takes place if a president must actively convince legislators to pass the policy. Whether or not learning occurs therefore depends on the president's proposed policy and the willingness of congress to oppose that policy. We show that the possibility of learning generates the following distortions in the bargaining process: First, it may be optimal for political players to oppose policies that they like. Second, congress may be unwilling to challenge a president, which could lead to president to promote more extreme policies.

We first consider the case where a non-contentious policy is followed by a contentious policy. While in the single-policy case a non-contentious policy always passes, in the dynamic case congress may have the incentive of attempting to block the policy. The rational for doing so is to weaken the president if he fails to get the policy passed. That is, if the belief about the president's ability is lowered, he would accommodate congress on the contentious policy issue. The incentive to weaken the president can be so strong that congress may oppose the president's first-period proposal even when its preferences over the first policy issue are identical to that of the president.

If the policy issues in both periods are contentious, it may also be the case that congress becomes much less willing to oppose the president. This happens if absent learning about ability, the president would accommodate congress in the second period. If congress were to oppose the president in the first period, and the president is successful in getting the policy passed despite the initial opposition, then it becomes more likely the president's ability is high. As a consequence, the president will be more aggressive in the second period and offer a less favorable policy to congress. However, if congress is unwilling to challenge the president in the first period, then the president can take advantage of congress' weakness, and propose more extreme policies in the first period. We show that in some cases the president's proposed policy can be more extreme than the president's ideal policy point.

Last, we show that these distortions can be minimized if "tough"(contentious) policy issues are first on the agenda, followed by "easy"(non-contentious) policy issues. Since the outcome of the non-contentious policy issue is the same regardless of the belief about the president's ability, there is no incentive to learn from the first policy issue, leading to no distortion in the outcome of the first policy issue. Moreover, we argue that the president would actually choose the tough policy as the first one if he is able to select the policy issue.

There is an extensive literature on legislative bargaining. As in Romer and Rosenthal (1979) and Diermeier and Fong (2011) we assume that there is a designated agenda setter. This assumption allows us to focus on learning about the type of only one of the players. In contrast, Baron and Ferejohn (1989) and Baron and Ferejohn (1987) consider the case of a decentralized com-

mittee in which each member can be selected to be the agenda setter. These models have been extended to multidimensional policy spaces by Banks and Duggan (2000) and Banks and Duggan (2006), and to sequential bargaining over different policies (c.f., Baron (1996), Kalandrakis (2004), Duggan and Kalandrakis (2012)). To link policies over time, these models assume that once a policy is enacted it determines that status quo point for the next period. In our model we do not specify a status quo point. Instead, if no agreement is reached then everyone receives a reservation utility, which in principle could be the utility from some policy that is already in place. However, we assume that the reservation utilities in our model are not affected by policy choices in previous period. The main reason for our assumption is that this leaves learning as the only inter-temporal link in the model, which therefore allows us to clearly identify the effects of learning in the legislative bargaining process.

Theoretically, the model of this paper contributes to the literature on learning and experimentation with many political agents. Strulovici (2010) shows that the inability to control the future policy choice leads each agent to vote conservatively in a policy reform experiment. Callander and Hummel (2014) consider a case in which a political party preemptively experiments on policy to affect future decisions of the opposition party. Our paper differs because in our model agents do not learn the type of the policy, but the strength of the president which endogenously determines the future outcome.

The existing literature on bargaining with incomplete information (Fudenberg et al., 1985; Abreu and Gul, 2000; Deneckere and Liang, 2006) typically focuses on the effect of private information of bargainer(s). In these models, a rejection by the informed bargainer signals that the bargainer has a higher reservation value. In contrast, in our paper players are symmetrically uninformed about the president's ability and their conflicts over policy induces social learning.

Using the definition of Mayhew (1991), gridlock refers to the ratio of the supply of policies to the demand for policy. In our model, the demand for policies is one in each period, and hence gridlock in each period corresponds to the probability that a policy is passed. Thus, our model also allows us to investigate the determinants of political gridlock. In a recent paper Ortner (2017) investigates gridlock in a dynamic model in which a player's bargaining position depends on a stochastic process that depends on past policy choices, referred to as the player's popularity. Gridlock arises as a consequence of a player's tradeoff between implementing the player's ideal policy and maximizing popularity. In contrast, in our model gridlock is solely driven by the president's innate bargaining ability and the player's incentives of learning about it.

Several papers in political agency literature consider cases in which career concerns of politicians could lead to "pandering" behavior (Prat, 2009; Fox and Van Weelden, 2010). In

our paper congress may be over accommodating to the president, but they do so in order to prevent strengthening the president.

## 2 Model

The model extends over  $T$  periods ( $t = 1, \dots, T$ ). The government consists of two separate political institutions, the president and congress. In each period, there is one exogenously given policy issue on the agenda. The president moves first, by announcing a policy proposal  $x_t \in X_t = [\underline{x}, \bar{x}]$ . Given proposal  $x_t$ , congress chooses whether or not to accept or oppose it, described by an action  $a \in \{y, n\}$ .<sup>1</sup> The president's ability is given by  $\omega \in \{\omega_h, \omega_l\}$ , where  $0 \leq \omega_l < \omega_h \leq 1$ . Both congress and the president are symmetrically uninformed about  $\omega$ . Let  $p_t$  be the common prior belief at time  $t$  that the president's ability level is high. The assumption that the president has the same information about his ability as congress applies if job experiences prior to being elected president that are not publicly observable do not help the president to be more effective in its interactions with congress.

The president's ability, as well as congress' response, impacts the probability that the proposed policy is implemented. If congress accepts the proposal, then the policy is implemented with probability one. If congress opposes the proposal, then the president can try to convince congress to change their position. The president's attempt succeeds with probability  $\omega$ , i.e., a more competent president is more likely to convince congress to pass a proposal. While we do not model the details of the process by which the president may convince members of congress to support controversial policies, one can assume that presidents can impose costs on member of congress who vote against the proposal, for example by encouraging primary competitors, or alternatively provide benefits by supporting reelection bids or providing pork to the members' district.

If the policy is not implemented, then the president and congress receive reservation utilities  $\underline{u}_{P,t}$  and  $\underline{u}_{C,t}$ , respectively, reflecting the utilities of not addressing the particular policy issue as a result of gridlock. For example, if the policy involves providing a public good, such as health insurance coverage under the ACA, then not passing the proposal means that zero units of that particular public are provided. In other words, the reservation utilities  $\underline{u}_{P,t}$ ,  $\underline{u}_{C,t}$  are the payoffs to the players if gridlock occurs.

If the policy is implemented then congress and the president receive utilities  $u_{C,t}(x) =$

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<sup>1</sup>The assumption that the president being the agenda-setter in every period is also imposed in Diermeier and Fong (2011). The literature has argued that in many political institutions, the elected president typically has a persistent agenda-setting power (Cheibub, 2007; Robinson and Torvik, 2016)

$v_t(\theta_{C,t}, x)$  and  $u_{P,t}(x) = v_t(\theta_{P,t}, x)$ , respectively, where  $v_t(\theta, x)$  is a period- $t$  utility of a player with preference type  $\theta \in \Theta$  from the policy position  $x$ .<sup>2</sup>

We introduce some assumptions on the utility function  $v_t$ .

**Assumption 1** *The utility  $v_t: \Theta \times X_t \rightarrow \mathbb{R}$  satisfies the following properties:*

1. *Strict concavity in  $x$ :  $\frac{\partial^2 v_t(\theta, x)}{\partial x^2} < 0$ ,*
2. *Single-crossing property:  $\frac{\partial^2 v_t(\theta, x)}{\partial \theta \partial x} > 0$ ,*

For the most results, it is sufficient to assume single-peaked property of  $v(\theta, \cdot)$  instead of strict concavity. The single-crossing property is only used for comparative statics with respect to  $\theta$ .

We consider perfect Bayesian equilibria of this game. A perfect Bayesian equilibrium is a profile in which (i) the president chooses an optimal action given the congress' strategy and his belief about his ability; (ii) congress responds to maximize its expected payoffs given the president's strategy and its belief about his ability; and (iii) the players' beliefs are updated via Bayes' rule whenever possible.

We conclude this section by providing two examples for the payoffs that fit the situation.

**Example 1** (Payoffs over a policy space) Suppose that the president proposes a policy that lies on a (ideological) policy space  $x \in [0, 1]$ . Congress and the president have their own ideal points, denoted by  $\theta_C$  and  $\theta_P$ , respectively, and utilities are  $u_P(x) = -(\theta_P - x)^2$  and  $u_C(x) = -(\theta_C - x)^2$ . Thus,  $v_t(\theta, x) = -(x - \theta)^2$ , which satisfies all of the above assumptions. ■

**Example 2** (Public good production) In each period consumers have a utility over private and public good consumption. Public goods,  $x$ , can be provided by the government, at the per-capita cost  $c(x)$ . In each period consumers are endowed with  $m$  units of the private good, and their utility is increasing in type  $\theta$ , i.e.,  $u_P(x) = \theta_P x - c(x)$  and  $u_C(x) = \theta_C x - c(x)$ . Thus,  $v_t(\theta, x) = \theta x - c(x)$ . If  $c'' > 0$  then condition 2 of Assumption 1 is satisfied. ■

### 3 Contentious and Non-Contentious Issues

We distinguish between policy issues that are easy to resolve and those that require at least one of the bargaining parties to compromise. More formally, Definition 1 below specifies that a

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<sup>2</sup>Note that we do not need to add discounting to utility because utility is already allowed to depend on  $t$ .

policy issue is not contentious if the ideal policy point of the president is also strictly better for congress than its reservation utility. Otherwise, we call that the policy issue is contentious.

**Definition 1** Let  $x_{P,t}^*$  be the president's most preferred policy, i.e.,  $x_{P,t}^* = \arg \max_{x \in X_t} u_{P,t}(x)$ .

The policy issue at time  $t$  is contentious if  $u_{C,t}(x_{P,t}^*) < \underline{u}_{C,t}$ . The policy issue is non-contentious if  $u_{C,t}(x_{P,t}^*) > \underline{u}_{C,t}$ .

If a policy issue is *too* contentious, then no policy position  $x$  would satisfy both parties. This is the case if the reservation utilities from not implementing policy ( $\underline{u}_P$  and  $\underline{u}_C$ ) exceed the individual's payoff for implemented proposal  $x$ . To exclude such trivial cases, we assume that for any policy issue, there exists at least one policy position  $x$  that makes both parties better off if the policy is implemented.

**Assumption 2** There exists  $x \in X_t$  such that  $u_{i,t}(x) > \underline{u}_{i,t}$  for  $i = P, C$  for any time  $t$ .

A situation in which the political players are unsuccessful in passing laws and addressing public policy problems is commonly referred to as *legislative gridlock*. Our model makes predictions about the level of gridlock that occurs and its dependence on the type of policy issues as well as the president's ability. It is therefore useful to recall a standard definition of gridlock used in the political science literature. In particular, Mayhew (1991) defines gridlock as the ratio of policies that are enacted (the supply of policies) divided by the policies that are on the agenda (the demand for policies).

In our model, there is one policy issue on the agenda in each period, and hence the demand for policies in each period  $t$  is one. The supply of policies is given by the probability  $q_t$ , that a policy is passed in period  $t$ . Thus, if  $q_t = 1$  there is no gridlock, and a lower value  $q_t$  corresponds to increased gridlock. Binder (1999) proposes a number of different hypotheses about the determinants of gridlock. In the following section we will revisit some of these hypotheses.

## 4 Equilibria in the One-Period Model

If the model extends only over one period ( $T = 1$ ) then there is no learning about the president's competency. As a consequence, congress will accept any proposal as long as the congress' utility of that proposal exceeds the payoff from not implementing a policy. Let  $x_P^* = \arg \max_{x \in X} u_P(x)$  be the president's most preferred policy position. Assumption 2 implies that the president is better off if this policy passes, i.e.,  $u_P(x_P^*) > \underline{u}_P$ .

If the policy issue is not contentious, i.e.,  $u_C(x_P^*) > \underline{u}_C$  then the unique equilibrium of the game involves the president proposing  $x_P^*$  which congress accepts. Thus, 100% of the policy agenda is passed and there is no gridlock.

Now suppose that the policy is contentious, i.e.,  $u_C(x_P^*) < \underline{u}_C$ . Without loss of generality suppose that  $x_P^* > x_C^*$ . Then the president has two options. First, propose a policy  $x$  with  $u_C(x) \geq \underline{u}_C$ . Strict concavity of the utility functions and Assumption 2 imply that in this case the president will select the unique policy  $\hat{x} \in [x_C^*, x_P^*]$  with  $u(\hat{x}) = \underline{u}_C$ . Alternatively, the president could choose a policy that congress dislikes but put pressure on congress to pass it nevertheless. As indicated in the model, the probability that this strategy succeeds depends on the president's ability. In particular, if  $p$  is the ex-ante probability that the president is competent, then the probability that the policy is passed is  $\omega(p) = p\omega_h + (1-p)\omega_l$ . The president's expected payoff is therefore  $\omega(p)u_P(x_P^*) + (1-\omega(p))\underline{u}_P$ . Thus, there exists  $\bar{p}$  such that the president chooses  $\hat{x}$  if  $p < \bar{p}$  and  $x_P^*$  for  $p > \bar{p}$ . In the first case there is no gridlock. In the second case, the probability that the policy passes is  $\omega(p)$ .

Therefore, as indicated in the right panel of Figure 1, gridlock is a non-monotone function of the president's expected ability. If the president's expected ability is low, the gridlock does not exist. Once probability  $p$  crosses the threshold,  $\bar{p}$ , gridlock is maximal, and starts to decrease as  $p$  increases. The intuition for this result is that a weak president only proposes policies that congress does not oppose, and hence gridlock is minimal. In contrast, as stronger president attempts to get more controversial policies passes, and the probability of passages increases in the president's ability.

The expected payoff to congress and to the president are shown in the left panel of Figure 1. Both are constant if  $p < \bar{p}$ . However, at the threshold  $\bar{p}$  the expected payoff of congress is discontinuous: With probability  $\omega(\bar{p})$  the president succeeds passing policy  $x_P^*$ , which gives congress a utility  $u_C(x_P^*) < \underline{u}_C$ . With the remaining probability,  $1 - p(\omega)$ , gridlock occurs and congress' payoff is  $\underline{u}_C$ .

Because of this discontinuity, there exists some policy  $x$  with  $u_C(x) < \underline{u}_C$  that results in a strictly higher payoff than the equilibrium payoff for  $p > \bar{p}$ . It follows immediately that the presidents also prefers this policy to the equilibrium policy if  $p$  is close to  $\bar{p}$ . In other words, the equilibrium with gridlock is Pareto inefficient. Proposition 1 shows that this inefficiency result holds for all  $p$  between  $\bar{p}$  and 1. Note, however, that if the president suggested policy  $x$ , then congress would reject it, because this would raise congress' utility. Knowing this, the president chooses  $x_P^*$  instead of  $x$ , generating the inefficient outcome.

If  $\omega_l$  is too large, then it becomes optimal even for a low ability president to challenge congress. Similarly, if  $\omega_h$  too small then the president will never challenge congress. In the

former case we would get  $\bar{p} = 0$ , and the latter case  $\bar{p} = 1$ .

An upper and lower bound for  $\omega_l$  and  $\omega_h$  which ensure that the cutoff  $\bar{p}$  is strictly between 0 and 1, can be derived as follows. First, it has to be the case that if the president's ability is low with certainty, i.e.,  $p = 0$ , then it is better for the president to accommodate. Thus,  $\omega_l v(\theta_P, x^*(\theta_P)) + (1 - \omega_l)\underline{u}_P < v(\theta_P, \hat{x})$ . Second, if the president is known to be of high ability, then it must be optimal for the president to challenge congress, i.e.,  $\omega_h v(\theta_P, x^*(\theta_P)) + (1 - \omega_h)\underline{u}_P > v(\theta_P, \hat{x})$ . Let

$$\bar{\omega} = \frac{v(\theta_P, \hat{x}) - \underline{u}_P}{v(\theta_P, x^*(\theta_P)) - \underline{u}_P}. \quad (1)$$

Then if  $\omega_l < \bar{\omega}$  it follows that it is optimal for the president to offer  $\hat{x}$  when  $p = 0$ . If  $\omega_h > \bar{\omega}$  and  $p = 1$  then it is optimal to offer  $x^*(\theta_P)$ . Note that  $0 < \bar{\omega} < 1$  because the numerator in (1) is the president's net-benefit from policy  $\hat{x}$ , while the denominator is the president's net benefit from his most preferred policy.

Let  $\bar{p}$  be the belief about the president's ability such that the president is indifferent between  $\hat{x}$  and  $x^*(\theta_P)$ . Then

$$\omega(\bar{p})v(\theta_P, x^*(\theta_P)) + (1 - \omega(\bar{p}))\underline{u}_P = v(\theta_P, \hat{x}).$$

Solving this equation for  $\bar{p}$  yields

$$\bar{p} = \frac{v(\theta_P, \hat{x}) - \underline{u}_P - \omega_l(v(\theta_P, x^*(\theta_P)) - \underline{u}_P)}{(\omega_h - \omega_l)(v(\theta_P, x^*(\theta_P)) - \underline{u}_P)},$$

which simplifies to

$$\bar{p} = \frac{\bar{\omega} - \omega_l}{\omega_h - \omega_l}. \quad (2)$$

Thus, the intermediate value theorem implies that there exists a value of  $p$ , strictly between zero and one, such that the president is indifferent between offering  $\hat{x}$  and  $x^*(\theta_P)$ . This is the cutoff value  $\bar{p}$ .

It should be noted that  $\bar{\omega}$  can be easily determined for our Examples 1 and 2.

**Example 1 (continued).** Consider the spatial model of Example 1. Here  $x^*(\theta_P) = \theta_P$ . Therefore  $v(\theta_P, x^*(\theta_P)) = 0$ . Suppose that  $\theta_P > \theta_C$ . Then  $\hat{x}$  is a value between  $\theta_P$  and  $\theta_C$  such that  $v(\hat{x}, \theta_C) = \underline{u}_C$ . Thus,  $\hat{x} = \theta_C + \sqrt{-\underline{u}_C}$ .<sup>3</sup> Therefore

$$\bar{\omega} = 1 + \frac{(\theta_P - \theta_C - \sqrt{-\underline{u}_C})^2}{\underline{u}_P}. \quad (3)$$

Note that  $\bar{\omega} < 1$  because  $\underline{u}_P < 0$ . In order for the issue to be contentious, we need  $-(x^*(\theta_P) - \theta_C)^2 < -\underline{u}_C$ , which is equivalent to  $\theta_P - \theta_C - \sqrt{-\underline{u}_C} > 0$ .

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<sup>3</sup>If  $\theta_P < \theta_C$  then  $\hat{x} = \theta_C - \sqrt{-\underline{u}_C}$ .

**Example 2 (continued).** Consider the public good provision Example 2 with  $\theta_P > \theta_C$ . Suppose that costs are  $c(x) = x^2$  and that the reservation outcome is that no public good is provided, i.e.,  $\underline{u}_C = \underline{u}_P = 0$ . Then  $x^*(\theta_P) = \theta_P/2$ , while  $\hat{x} = \theta_C$ . Thus,

$$\bar{\omega} = \frac{4(\theta_P\theta_C - \theta_C^2)}{\theta_P}. \quad (4)$$

The issue is contentious if and only if  $\theta_C x^*(\theta_P) - \theta_P^2 < 0$ , which is equivalent to  $\theta_C < \theta_P/2$ .

We now state our first result.

**Proposition 1** *Suppose that the model extends to a single period ( $T = 1$ ).*

1. *If a policy is not contentious, then the president's policy proposal is always passed by congress, and thus there is no gridlock.*
2. *Suppose that a policy is contentious. Let  $\bar{p} = (\bar{\omega} - \omega_l)/(\omega_h - \omega_l)$ , where  $\bar{\omega}$  is defined in (1). Then for any  $p < \bar{p}$ , the policy always passes and there is no gridlock. For any  $p > \bar{p}$ , probability of gridlock is  $1 - (p\omega_h + (1 - p)\omega_l)$ , which is strictly decreasing in  $p$ .*
3. *Gridlock is Pareto inefficient, i.e., if  $p > \bar{p}$  there exists a policy  $x(p)$  that makes both the president and congress strictly better off compared to the equilibrium payoffs.*

It is worth noting that in the one-period case gridlock is solely caused by the president's attempt of passing policies that make congress worse off. In other words, congress would agree to any policy  $x$  with  $u_C(x) \geq \underline{u}_C$ . We show below that this is no longer true in the multi-period case.

Proposition 1 also shows that gridlock is not Pareto efficient, and hence socially costly. In particular, there exists a policy,  $x$ , which would make both congress and the president strictly better off if that policy is passed compared to the equilibrium payoffs. Note, however, that congress' utility from  $x$  is lower than the utility of having no policy implemented. Thus, if the president proposed  $x$ , congress would oppose it, making the president worse off policy  $x_p^*$  is also opposed by congress, however, in expectations the president is better off with  $x_p^*$  compared to  $x$ .

We next proceed with comparative statics about the cutoff  $\bar{p}$  that determines whether or not gridlock occurs. If preferences of congress and the president on a policy issue are sufficiently close, then that issue becomes non-contentious, and as a consequence, Proposition 1 implies that there is no gridlock. One may therefore expect that making preferences closer, i.e., reducing

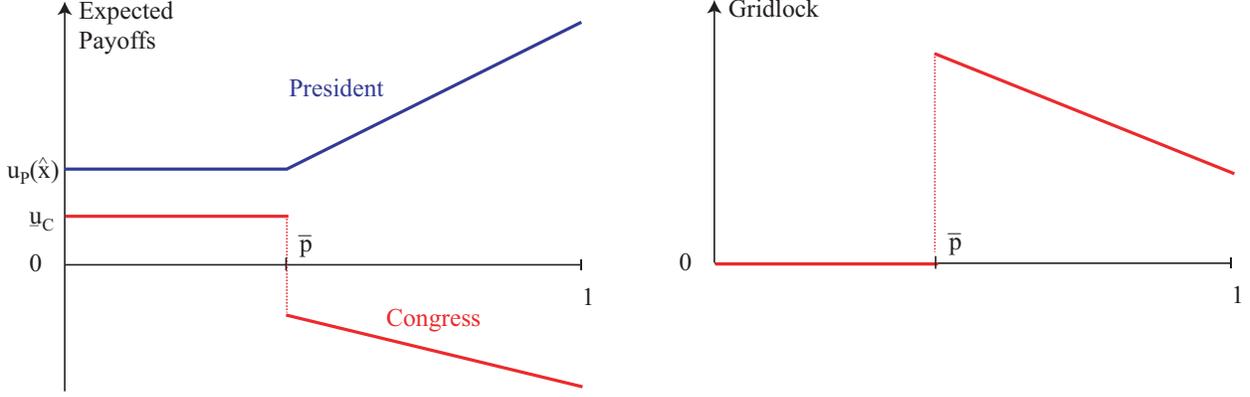


Figure 1: Expected Payoff Functions and Gridlock in One-period Model

$|\theta_P - \theta_C|$  reduces gridlock. However, this is not always the case as the following example indicates.

**Example 3** Consider a public good provision problem as in Example 2. In particular, assume that utilities are  $v(\theta, x) = \theta x - x^2$ . Let  $\theta_C = 0.25$ ,  $\theta_P = 2$ ,  $\underline{u}_C = 0$ , and  $\underline{u}_P = 0.4$ . Further, let  $\omega_l = 0$  and  $\omega_h = 1$ . Apart from the particular parameter choices, the only real difference to Example 2 is that the president's reservation utility is nonzero. Thus, president would rather have no policy passed than a very low level of public good provision.

Recall that if the president wants to accommodate congress, he proposes  $\hat{x} = \arg \max_{x \in X} v(\theta_P, x)$  subject to  $v(\theta_C, x) \geq \underline{u}_C$ . For our parameters we get  $\hat{x} = 0.25$ . A president who challenges congress proposes  $x_P^* = \arg \max_{x \in X} v(\theta_P, x) = \theta_P/2$ .

The cutoff belief,  $\bar{p}$ , at which the president is indifferent between accommodating and challenging congress is therefore given by  $\bar{p}(0.25\theta_P^2) + (1 - \bar{p})(0.4) = 0.25\theta_P - (0.25)^2$  and hence

$$\bar{p} = \frac{20\theta_P - 37}{20\theta_P^2 - 32}. \quad (5)$$

Thus, at  $\bar{p} = 1/16$  if  $\theta_P = 2$ . It is easy to check that the derivative of (5) with respect to  $\theta$  at  $\theta = 2$  is strictly positive. Thus,  $\bar{p}$  is increasing in  $\theta_P$ , which means that the set of parameters,  $p$ , for which gridlock occurs decreases as  $\theta_P$  is increased. For example, suppose  $p = 0.1$ , then if  $\theta_P = 2$ , the president challenges and there is a 10% probability of gridlock. However, if  $\theta_P$  is increased to 2.2, the president accommodates and the probability of gridlock is zero. ■

Intuitively, what happens in Example 3 is that raising  $\theta_P$  raises both the utility of policy  $\hat{x}$  that congress would not oppose, as well as the utility of  $x_P^*$ , the president's most preferred

policy. For the current value of  $\theta_P$  the president's utility of  $\hat{x}$  is not much higher than the utility  $\underline{u}_P$  of not implementing any policy. As  $\theta_P$  is increased, this utility difference increases at a larger percentage than the gain from choosing  $x_P^*$ , and hence, gridlock becomes more costly to the president.

If, instead, raising  $\theta_P$  lowers the utility of  $\hat{x}$ , then gridlock always increases. This is the case if the utility at  $x_P^*$  is independent of  $\theta$ . This is the case in example 1, when utility is  $v_i(\theta, x) = -(x - \theta)^2$ . We also show that keeping  $\theta_P$  fixed, while moving  $\theta_C$  closer to  $\theta_P$  raises  $\bar{p}$ , thus reducing the number of cases in which gridlock occurs.

Finally, if the cost of gridlock to the president is increased (i.e.,  $\underline{u}_P$  is decreased) then the president is less likely to cause gridlock. Because gridlock in the one-period case is caused by the president, gridlock becomes less likely if congress is more accommodating, which is the case if  $\underline{u}_C$  is lowered. We state this result formally.

**Proposition 2** *Suppose that there is only one period, i.e.,  $T = 1$  and that the policy issue is contentious. Suppose also that  $\omega_l < \bar{\omega} < \omega_h$ , so that  $\bar{p} \in (0, 1)$ . Then  $\bar{p}$  increases—hence gridlock occurs for a smaller range of  $p$ —if*

1.  $\underline{u}_C$  decreases or  $\underline{u}_P$  increases;
2.  $\theta_C$  is moved toward  $\theta_P$ , provided that  $\theta_C < \theta_P$  and higher types receive a higher utility from policies  $x$  that are larger than their optimal policy, that is, if  $\frac{\partial v_i(\theta, x)}{\partial x} < 0$  then  $\frac{\partial v_i(\theta, x)}{\partial \theta} > 0$ ;
3.  $\theta_P$  is moved toward  $\theta_C$ , provided that  $\theta_C < \theta_P$  and the president's maximum utility is nonincreasing in  $\theta_P$ , that is,  $\frac{d}{d\theta_P}(v(\theta_P, x^*(\theta_P))) \leq 0$ .<sup>4</sup>

The static version of our model could explain several of hypotheses about the determinants of gridlock proposed by Binder (1999). In the partisan and electoral context, our comparative statics result regarding the distance between  $\theta_C$  and  $\theta_P$  explains Binder (1999)'s hypothesis regarding the polarization (“Hypothesis 2: The greater the polarization of the partisan elite, the higher is the level of policy gridlock.”). In the policy context, the reservation utility of congress  $\underline{u}_C$  can be thought of the level of federal budget surplus or the public support for governmental action (“Hypothesis 7: The greater the federal budget surplus relative to outlays, the lower is the level of policy gridlock; Hypothesis 8: The greater the level of public support for governmental action, the lower is the level of policy gridlock.”)

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<sup>4</sup>For parts 2 and 3 of Proposition 2, the conditions are opposite for the case of  $\theta_C > \theta_P$ ; For example, for part 3, the condition also holds if  $\theta_C > \theta_P$  and the president's maximum utility is *nondecreasing* in  $\theta_P$

Furthermore, extending our model to more than a single period provides new insights regarding the informational incentives, which could increase or decrease the amount of gridlock. The dynamic version of the model will be analyzed in the next chapter.

## 5 Equilibria in a Two-Period Model

### 5.1 Gridlock for Non-Contentious Issues

We have seen that in a one-period model, gridlock can only occur for contentious issues. Further, given the president proposes policies, gridlock is caused by the president proposing policies which give congress less than the reservation utility  $\bar{u}_C$ . In other words, the president is the sole cause for gridlock. Both of these insights are not longer true in the two-period case.

Suppose we start in period 1 with a non-contentious policy issue, followed by a contentious policy issue in the second period. For heuristic purposes, suppose that congress and the president have the same preference for the first policy issue, i.e.,  $\theta_{P,1} = \theta_{C,1}$ . Let  $p_1$  be the prior belief of the president's ability.

Equilibrium in the second period is the same as one in the one-period model in the previous section. In the two-period model, the expected payoff functions in Figure 1 become the continuation payoff function  $V_{C,2}(p_2)$  and  $V_{P,2}(p_2)$  in the second period as a function of the posterior belief  $p_2$ . Denote the cutoff value for beliefs in the second period by  $\bar{p}_2$ . In this subsection, we consider the case with high prior belief about the president's ability—in particular, we assume  $p_1 > \bar{p}_2$ .

In the first period, it is optimal for the president to propose policy  $x_{P,1}^* = \arg \max_{x \in X} v_1(\theta_P, x)$ , which also maximizes the utility of congress given that preference parameters are the same. If congress accepts the policy, the prior about the president's ability does not change, i.e.,  $p_1 = p_2$ . Because  $p_1 > \bar{p}_2$  there is gridlock in the second period. Given the discontinuity of congress' payoff function at  $p_2$ , it may be optimal to ensure that the updated prior  $p_2$  is at least with some positive probability to the left of  $\bar{p}_2$ . Congress can achieve this by opposing the first-period policy, even though it is congress' most preferred policy. In other words, we may observe gridlock although the policy is non-contentious, and this gridlock is caused by congress.

Consider the following example.

**Example 4** Suppose that the set of policies in both periods is  $X = [0, 1]$ . Let  $v_i(\theta, x) = -(x - \theta)^2$ . In the first period, the ideal points are  $\theta_{P,1} = \theta_{C,1} = 1/2$ , and  $\theta_{P,2} = 1$ ,  $\theta_{C,1} = 0$  in the second period. Let  $\omega_l = 1/2$  and  $\omega_h = 1$ . The reservation utilities are  $\underline{u}_C = -1/4$  and  $\underline{u}_P = -1$  in both

periods. As defined above, let  $\omega(p) = p\omega_h + (1-p)\omega_l = (1+p)/2$  be the probability that the president manages to pass a policy initially opposed by congress.

Solving for the equilibrium in the subgames starting in the second period we get  $\hat{x}_2 = 1/2$ , and the president is indifferent between accommodating congress with  $\hat{x}_2$ , or offering his most preferred policy  $x_{p,2}^* = 1$  if  $\omega(\bar{p})(-x_{p,2}^* - 1)^2 + (1 - \omega(\bar{p}))\underline{u}_{p,2} = -(\hat{x}_2 - 1)^2$ , i.e., if  $\bar{p} = 1/2$ . Thus, the expected payoffs are

$$V_{P,2}(p) = \begin{cases} -\frac{1}{4} & \text{if } 0 \leq p < \frac{1}{2}; \\ -\frac{1}{2} + \frac{p}{2} & \text{if } \frac{1}{2} < p \leq 1; \end{cases} \quad V_{C,2}(p) = \begin{cases} -\frac{1}{4} & \text{if } 0 \leq p < \frac{1}{2}; \\ -\frac{5}{8} - \frac{3p}{8} & \text{if } \frac{1}{2} < p \leq 1. \end{cases}$$

Now suppose that  $p_1 > 1/2$ . In the first period, both players have the same preference parameter and the president proposes their most preferred policy  $x_{p,1}^* = 1/2$ . Suppose congress accepts the proposal. Then the policy passes with probability one, yielding utility of zero to congress. In the second period, the posterior stays the same ( $p_1 = p_2$ ), and thus the congress receives the expected payoff of  $V_{C,2}(p_1)$ .

If congress opposes the policy, then the proposal is passed with probability  $\omega(p_1) = (1 + p_1)/2$  (congress receiving utility 0), or it fails with the remaining probability of  $(1 - p_1)/2$  (and congress receives utility  $\underline{u}_C = -1/4$ ).

After the opposition of congress, the posterior belief  $p_2$  is updated depending on the first-period outcome. If the project fails to pass, then the fact that  $\omega_h = 1$  implies that the president's ability is low (that is,  $p_2 = 0$ ). As a consequence, the president will accommodate by proposing  $\hat{x}_2 = 1/2$ , which yields utility  $\underline{u}_C = -1/4$  to congress. If the policy passes in the first period against the opposition of congress, then the posterior belief becomes  $p_2 = 2p_1/(1 + p_1) > p_1$ . By assumption  $p_1 > 1/2$ , and hence the president will challenge congress with proposal  $x_{p,2}^* = 1$  and succeed with probability  $p_2$ . Congress receives a payoff of  $-1$  in this case. With probability  $1 - p_2$  proposal  $x_{p,2}^*$  does not pass and congress' payoff is  $\underline{u}_C$ .

Adding all these terms, it follows that congress' ex-ante expected payoff from opposing the policy in the first period is  $-(9 + 7p_1)/16$ . The net-benefit to congress from opposing the policy in the first period is therefore  $-(9 + 7p_1)/16 - V_{C,2}(p_1) = (1 - p_1)/16$ , which is positive for any prior  $p_1 < 1$ .

Gridlock occurs in the first period with probability  $(1 - p_1)/2$  when  $p > 1/2$  and with probability zero when  $p < 1/2$ . Again, we observe the non-monotonicity of gridlock discussed in proposition 3. However, the main difference is that gridlock occurs for an issue about which both congress and the president share the same preferences. Moreover, it is congress who causes gridlock.

In the second period, gridlock occurs only if the project succeeds in the first period but fails

in the second. The ex-ante probability of this event is  $(1 - p_1)/4$ . Thus, the controversy over the contentious issue in the second period is not just shifted to the first period. Instead, we get gridlock in both periods, with the incidence of gridlock in the first period on the non-contentious issue being the same as the level of gridlock in a one-period model where only the contentious issue is present. ■

We next investigate more generally when gridlock occurs for non-contentious issues. As in example 4 we assume that there is a non-contentious policy in the first period, followed by a contentious policy in the second period. To analyze equilibria of the game, we proceed by backward induction. The equilibria in the second period are the same as those in the one-period model characterized in proposition 1. We define  $\bar{\omega}_t$  for the second period as in (1), i.e.,

$$\bar{\omega}_t = \frac{v_t(\theta_{P,t}, \hat{x}_t) - \underline{u}_{P,t}}{v(\theta_{P,t}, x_t^*(\theta_{P,t})) - \underline{u}_{P,t}}, \quad (6)$$

where  $\hat{x}_t$  is the president's most preferred policy that gives congress its reservation utility in the second period, i.e.,  $\hat{x}_t = \arg \max_x v_t(\theta_{P,t}, x)$  s.t.  $v_t(\theta_{C,t}, x) \geq \underline{u}_{C,t}$  and  $x_t^*(\theta_{P,2})$  is the president's unconstrained most preferred policy, i.e.,  $x_t^*(\theta_{P,2}) = \arg \max_x v_t(\theta_{P,t}, x)$

A key parameter for characterizing equilibria in the two period model is given by

$$\alpha = \frac{v_1(\theta_{C,1}, x^*(\theta_{P,1})) - \underline{u}_{C,1}}{\underline{u}_{C,2} - v_2(\theta_{C,2}, x^*(\theta_{P,2}))}. \quad (7)$$

Note that  $\alpha$  is the ratio of congress' net-benefit from receiving the president's most preferred policy in the first period over the cost of the president's most preferred policy in the second period. Note that the numerator is strictly positive if the issues at  $t = 1$  is not contentious, while the denominator is strictly positive if the second period policy issue is contentious.

For example, if we consider the spatial voting model of example 1 then

$$\alpha = \frac{-(\theta_{C,1} - \theta_{P,1})^2 - \underline{u}_{C,1}}{\underline{u}_{C,2} + (\theta_{C,2} - \theta_{P,2})^2}. \quad (8)$$

In the the above example, gridlock occurs because it useful for congress to weaken the president. That is, if the president does manage to get the project passed in the first period, the updated prior is below the cutoff  $\bar{p}_2$ . Hence it becomes optimal for the president to accommodate congress. Therefore gridlock over the non-contentious issue requires the following: First, the initial belief  $p_1 > \bar{p}_2$ , and hence if no additional information is revealed the president will offer his most preferred policy,  $x_2^*(\theta_{P,2})$ . Second, if bad information about the president's ability is revealed, then the updated prior,  $\pi_F(p_1) < \bar{p}_2$ . Thus, in the first period, the belief  $p_1 < \hat{p}_1$  where

$$\bar{p}_2 = \pi_F(\hat{p}_1) = \frac{\hat{p}_1(1 - \omega_h)}{\hat{p}_1(1 - \omega_h) + (1 - \hat{p}_1)(1 - \omega_l)}. \quad (9)$$

If  $\omega_l < \omega_h$  then it is immediate that  $\hat{p}_1 > \bar{p}_2$ .

The following result characterizes the conditions under which gridlock occurs.

**Proposition 3** *Suppose there are two time periods  $T = 2$  and that the first-period policy is not contentious, while the second period policy is contentious. Further, let  $\omega_l < \bar{\omega}_2$  and  $\omega_h > \bar{\omega}_2$ , where  $\bar{\omega}_2$  is defined in (6). Let  $\bar{p}_2 = (\bar{\omega}_2 - \omega_l)/(\omega_h - \omega_l)$ , and  $\alpha$  be given by (7). Let  $\omega(p) = \omega_h p + \omega_l(1 - p)$ . Finally, let  $\hat{p}_1$  be defined by (9).*

*Then:*

1. *Gridlock occurs with probability  $1 - \omega(p_1)$  in the first period in the following cases:*

(a) *For all  $p_1$  with  $\bar{p}_2 < p_1 < \hat{p}_1$  if  $\omega_l \geq \alpha$ .*

(b) *For all  $p_1$  with  $\max\{\bar{p}_1, \bar{p}_2\} < p_1 < \hat{p}_1$ , if  $\omega_l < \alpha < \omega_h$ , where*

$$\bar{p}_1 = \frac{(1 - \omega_l)(\alpha - \omega_l)}{(1 + \alpha - \omega_h - \omega_l)(\omega_h - \omega_l)}. \quad (10)$$

*In all other cases there is no gridlock in the first period.*

2. *If  $p_1 < \bar{p}_2$  then there is not gridlock in the second period. If  $p_1 > \hat{p}_1$  then the probability of gridlock in the second period is  $1 - \omega(p_1)$ . If there is gridlock in the first period, then the ex-ante probability of gridlock in the second period is  $\omega(p_1)(1 - \omega(\pi_S(p_1)))$ , where  $\pi_S(p_1) = \omega_h p_1 / (\omega_h p_1 + \omega_l(1 - p_1))$  is the updated prior after the president fails to get the first-period policy passed.*

3. *Gridlock is Pareto inefficient, i.e., if gridlock occur in either period, then there exists policies in both periods that make the president and congress strictly better off.*

Consider again example 4. Equation (8) implies that  $\alpha = 1/3$ . Because  $\omega_l = 1/2 > \alpha$  the first case in proposition 3 applies. Because  $\hat{p}_1 = 0$  it follows that gridlock occurs for all  $p_1 > \bar{p}_2$ . Further, (3) implies that  $\bar{\omega}_2 = 3/4$ , and hence  $\bar{p}_2 = 1/2$ .

Now suppose we lower  $\omega_h$  to 0.9 and raise  $\omega_l$  to 0.6. This does not affect that values of  $\bar{\omega}_2$  and  $\alpha$ . Using the formula in proposition 3 it follows that the value of  $\bar{p}_2$  remains at 1/2. However, (9) implies that  $\hat{p}_1$  decreases to 4/5. Thus, gridlock no longer occurs if the president's expected ability is sufficiently high, i.e., if  $p_1 > \hat{p}_1$ .

In the second period, there is no gridlock if  $p_1 < \bar{p}_2$ . If  $p_1$  is between  $\bar{p}_2$  and  $\hat{p}_1$  then as stated in the proposition, gridlock occurs with probability  $1 - \omega(p_1) = 0.3 - 0.1p$  in the first period, and in the second period with probability  $p\omega_h(1 - \omega_h) + (1 - p)\omega_l(1 - \omega_l) = 0.21 - 0.05p_1$ . If  $p_1 > \hat{p}_1$  gridlock in the second period occurs with probability  $1 - \omega(p_1) = 0.3 - 0.1p$ .

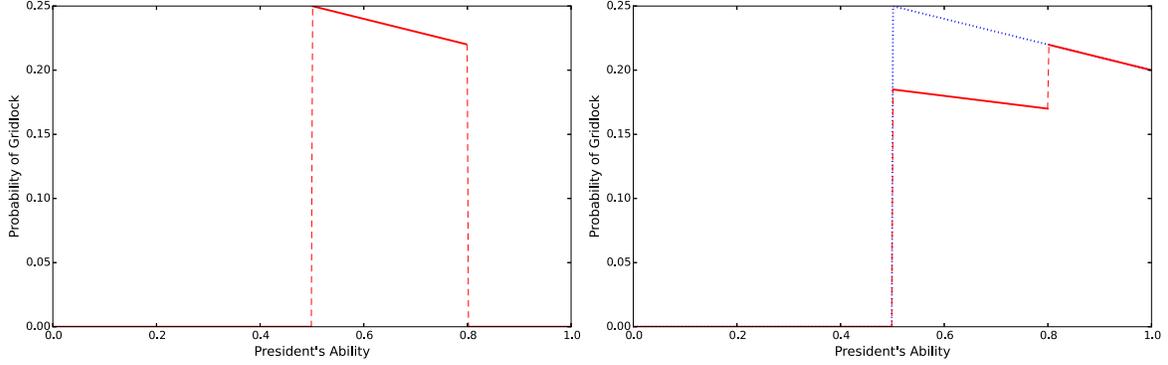


Figure 2: Gridlock in the first (left panel) and second period (right panel)

Figure 2 shows the incidence of gridlock in the first period (left panel) and second period (right panel) as a function of the president's ability  $p_1$ . As in the one period case shown in Figure 1 gridlock is a non-monotone function of  $p_1$ , and no gridlock occurs if the president's ability is low. However, unlike in the one-period case, the left panel indicates that no gridlock occurs in the first period if the president's ability is sufficiently high. This is because if the president is initially too strong, learning from the first policy issue does not affect his future behavior.

The dashed blue line in the right panel shows that level of gridlock that would occur if there were no first period, i.e., if we had a one-period model. Comparing that dashed line to the solid line we can see the impact of the first period on the second model period. That is, we get some reduction of gridlock if  $p_1$  is between 0.5 and 0.8, but this reduction does not make up for the increase of gridlock in the first period. The reason why gridlock occurs in the first period is because of the congress' incentive of weakening the president. For example, if these incentives were not present, and  $p_1 = 1/2$  (or more precisely is marginally larger than  $1/2$ ) we would have 0 gridlock in period 1 and 25% gridlock in period 2, i.e., an average rate of gridlock of 12.5% over the two periods. In contrast, in equilibrium we have a rate of 25% in the first period, and of 18.5% in the second period, i.e., an average rate of 21.75%.

Proposition 3 indicates that in the first period gridlock is linearly decreasing in  $p_1$  between  $\bar{p}_2$  and  $\hat{p}_1$ , and hence the left-panel of figure 2 shows the generic case. The gridlock curve between  $p_1$  and  $\bar{p}_2$  for the second period is also a straight line, but the slope need not be negative as in the right-panel of figure 2.

As indicated in proposition 3, gridlock in the second period is determined by two factors: The first factor is the probability  $\omega(p_1)$  that the president is successful in the first period, which is an increasing function of  $p_1$ . The second factor is the probability of the president not being

successful in the second period, contingent on being successful in the first period, which is given by  $1 - \omega(\pi_S(p_1))$ . This second factor is decreasing in  $p_1$ . It is easy to verify that  $\omega(p_1)(1 - \omega(\pi_S(p_1))) = p_1(\omega_h - \omega_l)(1 - \omega_h - \omega_l) - \omega_l(1 - \omega_l)$ . Thus, the slope of the line is positive, and gridlock increases with  $p$  if  $\omega_l + \omega_h < 1$ .

For example, suppose  $\theta_{C,1} = 0.05$ ,  $\omega_l = 0.1$  and  $\omega_h = 0.8$ . Let  $\underline{u}_p = -0.5$  with all other parameters being the same as in example 4. It follows immediately that  $\alpha = 0.633$  and that  $\bar{\omega}_2 = 0.5$ . Thus, the proposition implies that  $\bar{p}_2 = 0.571$ . Further  $\hat{p}_1 = 0.857$ . Then the probability of gridlock in the second period between  $\bar{p}_1$  and  $\hat{p}_1$  is  $0.09 + 0.07p_1$ , which is increasing in  $p_1$ .

Finally, proposition 3 shows that gridlock remains inefficient in the two-period model. For the first model period this is immediate. The president proposes his most preferred policy  $x^*(\theta_{P,1})$  in the first period, which makes congress strictly better off than  $\underline{u}_{C,1}$ . If congress opposes the policy, then congress' payoff is  $\underline{u}_{C,1}$  with probability  $1 - \omega(p_1)$  and  $x^*(\theta_{P,1})$  with the remaining probability. Thus, congress' utility in the first period is strictly decreased in the gridlock case. The argument for the second period is the same as in proposition 1.

We next investigate what happens if issues become more or less contentious.

**Proposition 4** *Suppose there are two time periods  $T = 2$  and that the first-period policy is not contentious, while the second period policy is contentious. Then the set of parameters,  $p_1$ ,  $\omega_b$ ,  $\omega_h$  for which gridlock occurs about the non-contentious issue increases if  $\theta_{C,1}$  is moved away from  $\theta_{P,1}$  or  $\theta_{C,2}$  is moved away from  $\theta_{P,2}$ .*

Intuitively, if the second period policy issues becomes more contentious, in the sense that the difference in preferences increases, then gridlock becomes more likely: The benefit to congress of generating gridlock is to weaken the president, and that benefit increases if the difference in preferences is increased. Similarly, if we increases the difference in preferences in the first period, then the net benefit to congress from the first policy issue, and hence the cost of gridlock decreases. As a consequence, gridlock occurs for more parameter values.

## 5.2 Accommodating Congress for Contentious Issues

For the static model, Proposition 1 shows that gridlock can only if an issue is contentious. For contentious issue gridlock always occurs if  $\omega_h$  is not too small and it is sufficiently likely that the president's ability is high. In this case the president proposes his most preferred policy  $x_P^*$ , which is rejected by congress. Moreover, congress would reject any policy that yields a lower payoff than  $\underline{u}_C$ .

We now investigate how the incentives and congress change in the dynamic model. We start, by providing an example in which the incidence of gridlock is reduced, because congress is unwilling to oppose the president.

**Example 5** Consider again the spatial voting model of example 4. Utilities are  $v_i(\theta, x) = -(x - \theta)^2$  on the set of policies  $X = [0, 1]$  in both periods. Suppose that the ideal points of the president are  $\theta_{P,1} = \theta_{P,2} = 1$  while those of Congress are  $\theta_{C,1} = 1/4$  and  $\theta_{C,2} = 1/2$ . The reservation utilities are  $\underline{u}_C = -1/16$  and  $\underline{u}_P = -1$  in both periods. Finally, suppose that  $\omega_l = 0$  and  $\omega_h = 1$ .

Note that the policies in both periods are contentious: The president's ideal policy is  $x_p^* = 1$ , and congress' utility from these policies are  $-1$ , and  $-1/4$ , respectively, which is less than  $\underline{u}_C$ .

We can determine equilibria in the second period by using Proposition 1. In particular, (3) implies that  $\bar{\omega}_2 = 15/16$ . Further,  $\bar{p}_2 = (\bar{\omega} - \omega_l)/(\omega_h - \omega_l) = \bar{\omega}$ , i.e., gridlock occurs in the second period if the prior about the president's ability in the second period is  $p_2 > 15/16$ .

If congress and the president were myopic in the first period then we can determine the cutoff for  $p_1$  that gives rise to gridlock by using Proposition 1 and (3) imply that  $\bar{\omega}_1 = 3/4$ , and hence  $\bar{p}_1 = 3/4$ . However, we now show that because of the strategic considerations in the dynamic game (when players are not myopic), gridlock does not occur for priors  $p_1$  between  $\bar{p}_1$  and  $\bar{p}_2$ .

The president's most preferred policy that gives congress its reservation utility in the second period is  $\hat{x}_2 = 3/4$ . This is the policy the president offers if  $p_2 < \bar{p}_2 = 15/16$ . If  $p_2 > \bar{p}_2$  then the president offers policy  $x^* = 1$ . In the first period, both players need to take into account the effect of policy outcomes on the posterior belief of the president's ability. Suppose that the congress rejects the president's proposal. Given that  $\omega_l = 0$ , passage of the policy means that the president is of high ability with probability 1, i.e.,  $p_2 = 1$ . If the project fails, then the updated prior  $p_2 = 0$ .

Suppose the president makes an offer  $x_1 = 1$ . Congress' payoff if it accepts the proposal  $-(1 - \theta_{C,1})^2 + (\hat{x}_1 - \theta_{C,2})^2 = -5/8$ . Now suppose that congress opposes the proposal. A president who gets the proposal passed is of high ability, i.e.  $p_2 = 1$ , and of low ability,  $p_2 = 0$ , otherwise. Thus, congress' expected payoff is

$$p_1 \left( -(1 - \theta_{C,1})^2 - (1 - \theta_{C,2})^2 \right) + (1 - p_1) \left( \underline{u}_C - (\hat{x}_2 - \theta_{C,2})^2 \right) = -\frac{1}{8} - \frac{11p_1}{16}.$$

Thus, accepting  $x_1 = 1$  is better for congress if  $p_1 > 11/18$ . Thus, congress will accommodate the president for all  $p_1$  with  $\bar{p}_1 < p_2 < 15/16$ , and hence gridlock is reduced. However, also notice that the president will choose policy  $x_1 = 1$  even if  $11/18 \leq p_1 < \bar{p}_1$ . In other words, the president takes advantage of congress' unwillingness to oppose even extreme policies. ■

What drives example 5 is that disagreement in the second period is larger than in the first period. At first glance this may look like a similar situation as in example 4 but there are two important differences. First, the policy in the period one is contentious, and hence congress would be willing to oppose the president in a one-period setting (in our case if  $p_1 > 3/4$ ). Second, the initial prior  $p_1 < \bar{p}_2$ . Hence, if no information is revealed then the president will accommodate congress in the second period by choosing the more moderate proposal  $\hat{x}_2 = 3/4$  instead of  $x^* = 1$ .

In this setting information revelation is bad for congress - if the president is revealed to be weak, decisions in the second period are unaffected, but if the president wins in the first period, the president will challenge congress in the second period with the extreme policy  $x^* = 1$ . As a consequence, congress is unwilling to confront the president, and the president takes advantage of it. For example, in a one-period model the president would choose policy  $x_1 = 1/2$  because  $-(1/2 - \theta_{C,1})^2 = \underline{u}_C$ , i.e., congress is indifferent between accepting and rejecting  $1/2$ . However, in the two period-model the president can offer  $x_1 = 1$  without any opposition by congress. This outcome is certainly efficient because gridlock never occurs, but from a utilitarian viewpoint it is not desirable.

To see this, again suppose that  $p_1 = 11/18$ . As we have shown, we get policies  $x_1 = 1$  in period 1 and  $x_2 = 3/4$  in period 2. If we weigh the utilities of congress and the president equally, then the utilitarian maximum is obtained by choosing  $x_1 = 5/8$  and  $x_2 = 3/4$ . In other words, the first period policy in the equilibrium of the model is too extreme.

We next investigate the issues discussed in example 5 more generally. As mentioned the policy issues in both periods must be contentious. Further, we consider initial beliefs  $p_1 < \bar{p}_2$ , where  $\bar{p}_2$  is the second period belief cutoff defined in section 5.1, i.e.,  $\bar{p}_2 = (\bar{\omega}_2 - \omega_l)/(\omega_h - \omega_l)$  where  $\bar{\omega}_2$  is defined in (6).

If the first-period policy passes unopposed, then gridlock never occurs in the second period, because there is no opportunity to learn about the president's ability if congress does not oppose the president's proposal, i.e.,  $p_2 = p_1$  and because  $p_1 < \bar{p}_2$  the president accommodates congress in the second period.

If, instead, the first-period policy passes against initial opposition from congress, then Bayes' rule implies that the posterior belief in the second period becomes  $\pi_S(p_1) = p_1\omega_h/(p_1\omega_h + (1 - p_1)\omega_l)$ . Because  $\omega_h > \omega_l$  the updated belief  $\pi_S(p_1) > p_1$ . If  $\pi_S(p_1) > \bar{p}_2$  then the president will challenge congress in the second period with policy  $x_p^*$  and gridlock occurs with positive probability. Let  $\tilde{p}_1$  be the value for which  $\pi_S(\tilde{p}_1) = \bar{p}_2$ , i.e.,

$$\tilde{p}_1 = \frac{\bar{p}_2\omega_l}{(1 - \bar{p}_2)\omega_h + \bar{p}_2\omega_l}. \quad (11)$$

If congress is worse off if the updated belief rises above  $\bar{p}_2$  then congress becomes more accommodating. This, in turn, means that the president can make more extreme policy proposals without facing opposition by congress. Proposition 5 shows conditions under which congress is willing to accept policy that result in a lower payoff than the reservation utility. We also provide conditions such that similar to the example, congress is willing to accept the president's most preferred policy  $x_1^*(\theta_{P,1})$  in the first period, although the policy issue is contentious. The main condition for the latter results involves the ratio of payoffs from the first and second period of the president's most preferred policy. In particular, define

$$\beta_P = \frac{v_1(\theta_{P,1}, x_1^*(\theta_{P,1})) - \underline{u}_{P,1}}{v_2(\theta_{P,2}, x_2^*(\theta_{P,2})) - \underline{u}_{P,2}}, \text{ and } \beta_C = \frac{\underline{u}_{C,1} - v_1(\theta_{C,1}, x_1^*(\theta_{P,2}))}{\underline{u}_{C,2} - v_2(\theta_{C,2}, x_2^*(\theta_{P,2}))}. \quad (12)$$

The values of  $\beta_P$  and  $\beta_C$  can be easily determined for the spatial model of example 1 or the public good provision problem in example 2. In the spatial model,  $x_i^*(\theta_{P,t}) = \theta_{P,t}$ . Thus,  $\beta_P = \underline{u}_{P,1}/\underline{u}_{P,2}$  and  $\beta_C = (\underline{u}_{C,1} + (\theta_{C,1} - \theta_{P,1})^2)/(\underline{u}_{C,2} + (\theta_{C,2} - \theta_{P,2})^2)$ . In the public good provision model,  $x_i^*(\theta_{P,t}) = \theta_{P,t}/2$ . Thus, if  $\underline{u}_{P,t} = \underline{u}_{C,t} = 0$ , the  $\beta_P = \theta_{P,1}^2/\theta_{P,2}^2$  and  $\beta_C = (4\theta_{C,1}\theta_{P,1} - \theta_{P,1}^2)/(4\theta_{C,2}\theta_{P,2} - \theta_{P,2}^2)$ .

The next proposition summarizes the above analysis.

**Proposition 5** *Suppose there are two time periods  $T = 2$  with contentious policy issues in period. Let  $\bar{\omega}_t$ ,  $t = 1, 2$  be given by (6),  $\tilde{p}_1$  by (11) and  $\bar{p}_t = (\bar{\omega}_t - \omega_l)/(\omega_h - \omega_l)$ ,  $t = 1, 2$ . Let  $\omega_l < \bar{\omega}_2$  and  $\omega_h > \bar{\omega}_2$ .*

1. *Let  $p_1 \in (\tilde{p}_1, \bar{p}_2)$ . Then in the first period congress is willing to accept policy proposals  $x_1$  that give congress a payoff below its reservation utility, i.e., congress is willing to accept any  $x_1$  with  $v_1(x_1, \theta_{C,1}) \geq \tilde{u}_{C,1}(p_1)$ , where*

$$\tilde{u}_{C,1}(p_1) = \underline{u}_{C,1} - \frac{\omega_h^2 p_1 + \omega_l^2 (1 - p_1)}{1 - \omega_h p_1 - \omega_l (1 - p_1)} \left( \underline{u}_{C,2} - v_2(x_2^*(\theta_{P,2}), \theta_{C,1}) \right) < \underline{u}_{C,1} \quad (13)$$

2. *Suppose that*

$$\beta_P > \frac{(\omega_h - \bar{\omega}_2)(\bar{\omega}_2 - \omega_l)}{1 - \bar{\omega}_2}, \text{ and } \beta_C < \frac{(\omega_h - \bar{\omega}_2)(\bar{\omega}_2 - \omega_l) + \bar{\omega}_2^2}{1 - \bar{\omega}_2}. \quad (14)$$

*Then there exists  $\varepsilon > 0$  such that the president offers his most preferred policy,  $x_1^*$ , which congress does not oppose for all  $p_1 \in (\bar{p}_2 - \varepsilon, \bar{p}_2)$ .*

The first statement of the proposition shows that congress is willing to adopt a policy it dislikes in the first period, because of concerns of strengthening the president otherwise. However, in

equilibrium it may still be better for the president to challenge congress with a policy that results in a strictly lower utility than  $\tilde{u}_{C,1}(p_1)$ . The second statement provides conditions under which  $\tilde{u}_{C,1}(p_1)$  becomes sufficiently low such that congress is willing to accept even the president's unconstrained optimal policy  $x_1^*$ . Note that in addition to a condition constraining  $\beta_C$ , which ensures that congress is willing to accept  $x_1^*$ , (14) also contains a condition on the president, via parameter  $\beta_P$ . First intuition may suggest that such a condition is not needed because congress is already willing to accept the president's most preferred policy. However, what this argument overlooks is that the president's optimal policy in the static model, may no longer be optimal in the dynamic model.

Recall from the first statement of the proposition that congress is willing to accept extreme policies out of the concern that opposing the president would ultimately strengthen him. However, this also means that the president has an incentive of making policy offers that are so extreme that congress has no choice but to oppose them. This cannot happen in example 5 because we assumed that the policy space is limited to  $[0, 1]$  and the president's most extreme policy,  $x_1 = 1$  is also his most preferred policy. However, even if we extend the policy space, and allow the president to select policies  $x_1 > 1$ , the example still works. This can be easily seen by verifying the conditions in (14).

Recall that  $\bar{\omega}_2 = 15/16$ . Further,  $\beta_P = 1$ . Because  $\omega_l = 0$ , and  $\omega_h = 1$  we get  $\beta_P > (\omega_h - \bar{\omega}_2)(\bar{\omega}_2 - \omega_l)/(1 - \bar{\omega}_2) = \bar{\omega}_2 = 15/16$ . Similarly,  $\beta_C = 8/3$  and the right-hand side of the inequality in (14) is 15.

In order to find a situation in which the president would choose an extremist policy, we must lower  $\beta_P$  sufficiently, i.e., the president's surplus from the second period policy must be much larger than the surplus from the first period.

**Example 6** Consider again a spatial model as in example 5 but assume that  $\theta_{C,1} = 3/10$ ,  $\theta_{C,2} = 0$ ,  $\theta_{P,1} = 7/10$  and  $\theta_{P,2} = 1$ . The reservation utilities are  $\underline{u}_{C,1} = \underline{u}_{C,2} = -1/16$  for congress, and  $\underline{u}_{P,1} = -1/16$ ,  $\underline{u}_{P,2} = -1$  for the president. We again assume that  $\omega_l = 0$  and  $\omega_h = 1$ .

Note that both policy issues are contentious, but the ratio of net-benefits to the president from his ideal policies in both periods  $\beta_P = 1/16$ , which is lower than in example 5.

(3) implies that  $\bar{\omega}_2 = 7/16$ . Thus,  $\bar{p}_2 = (\bar{\omega}_2 - \omega_l)/(\omega_h - \omega_l) = \bar{\omega}_2 = 7/16$ .

Now consider (14). Note that  $(\omega_h - \bar{\omega}_2)(\bar{\omega}_2 - \omega_l)/(1 - \bar{\omega}_2) = 7/16$ . Because  $\beta_P = 1/16$  it follows that the first inequality in (14) is violated, i.e., it is not optimal for the president to offer  $x_2^* = 1$ . In contrast, the second inequality holds because  $\beta_C = 13/125$  and the right-hand side is  $7/9$ . Thus, congress is willing to accept  $x_1^*$  in the first period, but the president is not willing to offer it, because he is better off if the congress opposes his policy.

The policy  $x$ , at which congress is indifferent between accommodating and opposing congress

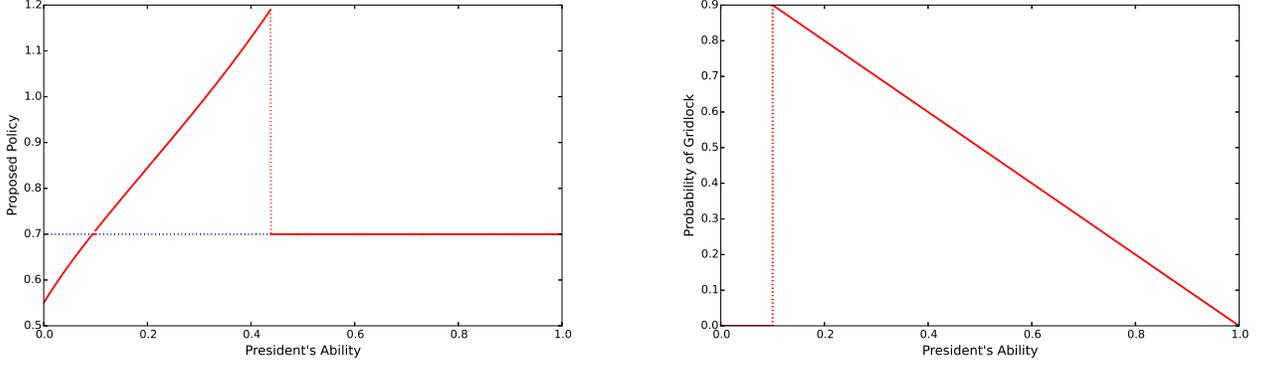


Figure 3: Proposed policy (left panel) and gridlock (right panel) in the first period

for some  $p_1 < \bar{p}_2$  satisfies

$$-(x - \theta_{C,1})^2 - (\hat{x}_2 - \theta_{C,2})^2 = p_1(-x - \theta_{C,1})^2 - (1 - \theta_{C,1})^2 + (1 - p_1)(\underline{u}_{C,1} - (\hat{x}_2 - \theta_{C,2})^2),$$

which implies

$$x = \frac{3}{10} + \frac{\sqrt{1 + 13p_1 - 14p_1^2}}{4(1 - p_1)} \quad (15)$$

Note that  $x > x_1^* = 0.7$  if  $p_1 > 13/138$ .

It is easy to check that the president is better off offering policy  $x$  instead of policy  $x_1^* = 0.7$  if  $p_1 \in (0.10001, \bar{p}_2)$ . In particular, the net-benefit of the president from policy  $x$  is

$$\begin{aligned} & p_1(-x - \theta_{P,1})^2 - (1 - \theta_{P,1})^2 + (1 - p_1)(\underline{u}_{P,1} - (\hat{x}_2 - \theta_{P,2})^2) - (-(x_1^* - \theta_{P,1}) - (\hat{x}_2 - \theta_{P,2})) \\ & = \left(\frac{5}{8} - \left(x - \frac{7}{10}\right)^2\right)p_1 - \frac{1}{16} \end{aligned} \quad (16)$$

One can check that (16) is strictly positive for all  $p_1 \in (0.10001, \bar{p}_2)$ . Thus, for all such values of  $p_1$  the president will choose policies that are more extreme than his ideal point. Congress opposes the policy, which leads again to gridlock. ■

Note that it is the president rather than congress who causes the gridlock in example 6. This happens despite the fact that congress is willing to accommodate, and even except policy  $x_1^*$ , despite the fact that congress' utility from  $x_1^*$  is below the reservation utility  $\underline{u}_{C,1}$ . This weakness of congress, in turn, induces the president to choose extremist policies. The left-panel of figure 3 shows the policy proposed by the president in the first period as a function of ability  $p_1$ . The president's ideal point is 0.7, and policy  $x_1^* = 0.7$  is proposed for all sufficiently large  $p_1$ . For intermediate values of  $p_1$  the proposed policy in fact exceeds 0.7. If the policy fails to pass,

i.e., if gridlock occurs, then the loss of the president is small because  $\beta_P$  is small. If the policy passes, then the president is strengthened in the second period. The updated prior  $p_2 > \bar{p}_2$  and the president will attempt to get his second period ideal policy  $x_2^*$  passed. In other words, offering the extreme policy in the first period gives the president the opportunity to test his ability to convince members of congress to vote for his proposals. If the president discovers that he is able to do this, then he will use this knowledge to his advantage in the future to get his agenda passed without trying to moderate.

The right panel of figure 3 shows the level of gridlock in the first period. If there were no second model period, then  $\bar{w} = \bar{p}_1 = 0.64$ , i.e., no gridlock would occur for  $p_1 \leq 0.64$ . However, because the president selects extreme policies for intermediate values of  $p_1$ , the set of parameters for which gridlock occurs is increased compared to the one-period case.

## 6 Agenda Selection

Assume that there exist two policy issues, and that one policy issue is contentious while the other is non-contentious. Now consider a case in which the president can choose the order of the policy agenda in each period. Which of the two policies would the president want to address first?

Suppose the president wants to address the non-contentious policy first. This leads to the case discussed in Subsection 5.1. That is, despite the fact that the first issue is not-contentious, congress may have the incentive of opposing the president in order to weaken him in the second period. While the president benefits from learning about his ability in such a case, this benefit is not outweighed by the cost generated by gridlock if the president's reservation utility in the first period is sufficiently low.

Now suppose that the president first selects the contentious issue. Then independent of what happens in the first period, in the second period the president will offer  $x_2^*$ , which congress accepts (because the second-period policy is not-contentious,  $x_2^*$  yields a payoff that is strictly higher than congress' reservation utility  $\underline{u}_{C,2}$ ). Thus, the policy decision for the non-contentious issue is no longer distorted.

If the president has control over the agenda, it is therefore optimal to choose the "difficult" issue, (i.e., the contentious issue) first. Choosing the "easy" issue last, avoids gridlock on these issues.

## 7 Concluding Remarks

The paper introduces learning about the agenda setter's ability into a legislative bargaining model. That is, the agenda setter, referred to as the president, has the power to pressure legislators (congress) to change their vote with some probability, but this ability is initially not known by participants. Learning occurs when the president attempts to use his power to persuade legislators.

We show that intertemporal incentives exist in a number of interesting ways, because of learning changes the bargaining outcome. First, even policy issues that are easy to resolve absent learning can become contested. This happens because opposing the president reveals information about the president's ability, and if the president's expected ability is lowered, the president has to compromise more in the future. Of course, if the president is able to get a policy passed against congress' initial opposition, then the president's expected ability is increased. This, in turn, means that the president will be more confrontational in the future, and we characterize situations in which this latter effects dominates the former, and hence congress appears weak and unwilling to confront the president. Finally, we show that if the president can select the order in which the address policy issues, then it is better to start with the difficult issues, that require compromise, as the first agenda.

One main points of departure of our model from the standard bargaining literature is the inclusion of a new aspect, bargaining ability. Bargaining ability measures the innate ability of a negotiator to convince their opponent that it is in their interest to accept a deal that is offered. *Ceteris paribus*, a higher ability individual is able to obtain a higher surplus in the bargaining process. In contrast, in classic bargaining models it is a first mover advantage as well as a bargainer's patience that matters.<sup>5</sup>

In our model the bargaining ability is fixed, and individuals learn about it when they interact with each other. Further, we assume that only the agenda setter's ability matters. It would be interesting to extend the present model to allow for both learning about the parties abilities, as well as allowing for the possibility that individual can improve their bargaining skill through experience.

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<sup>5</sup>In a related paper, Lee and Liu (2013) consider a case in which a bargainer privately knows his type, which affects an exogenous disagreement payoff. They analyzed the effect of private information of the bargainer on the dynamics of bargaining outcome.

## 8 Appendix

**Proof of Proposition 1.** The first two statements are shown in the text. It remains to prove that gridlock is inefficient.

Note that there is no gridlock if  $x_p^* = x_c^*$ . Thus, let  $x_p^* \neq x_c^*$ . Without loss of generality, assume that  $x_p^* > x_c^*$ . Further, for gridlock to occur the issue must be contentious, i.e.,  $u_C(x_p^*) < \underline{u}_C$ .

Because of strict concavity of utility, there exist functions  $x_i(q)$  with  $x_c^* \leq x_i(q) \leq x_p^*$  that solve  $u_i(x_i(q)) = qu_i(x_p^*) + (1 - q)\underline{u}_i$ , for  $i = P, C$ . Note that

$$x_i'(q) = \frac{u_i(x_p^*) - \underline{u}_i}{u_i'(x_i(q))}, \text{ and } x_i''(q) = -\frac{1}{u_i'(x_i(q))u_i''(x_i(q))}. \quad (17)$$

Let  $f(q) = x_C(q) - x_P(q)$ . We prove that  $f(q) > 0$  for  $0 < q < 1$ .

For  $0 < q < 1$  we have  $x_c^* < x_i(q) < x_p^*$  and hence  $u'_C(x_C(q)) < 0$  and  $u'_P(x_P(q)) > 0$ . Thus,  $u_i'' < 0$  for  $i = P, C$  and (17) imply that  $f''(q) < 0$ .

Next,  $f(0) > 0$ . In particular, assumption 2 implies that there exists  $\hat{x}$  with  $u_C(\hat{x}) = \underline{u}_C$  and  $u_P(\hat{x}) > \underline{u}_P$ . Then  $x_C(0) = \hat{x}$  and  $x_P(0) < x_C(0)$ . Further,  $f(1) = 0$  because  $x_C(1) = x_P(1) = x_p^*$ . Strict concavity of  $f$  therefore implies that  $f(q) > 0$  for all  $0 < q < 1$ .

We have therefore shown that  $x_C(q) > x_P(q)$ . Choose  $x$  with  $x_C(q) > x > x_P(q)$ . Then  $u_C(x) > u_C(x(q))$  and  $u_P(x) > u_P(x(q))$ , i.e., receiving policy  $x$  with certainty makes both players strictly better off. ■

**Proof of Proposition 2.** Suppose that the policy issue is contentious and that  $\omega_l < \bar{\omega} < \omega_h$ , where

$$\bar{\omega} = \frac{v(\theta_P, \hat{x}) - \underline{u}_P}{v(\theta_P, x^*(\theta_P)) - \underline{u}_P}. \quad (18)$$

Then Proposition 1 implies that the threshold belief  $\bar{p}$  about the president's ability is given by  $\bar{p} = (\bar{\omega} - \omega_l)/(\omega_h - \omega_l)$ .

1. If  $\underline{u}_C$  decreases, then  $v(\theta_P, \hat{x})$  increases since  $\hat{x} = \arg \max v(\theta_P, x)$  subject to  $v(\theta_C, x) \geq \underline{u}_C$ . Therefore, from (18),  $\bar{\omega}$  (and thus  $\bar{p}$ ) increases in  $\underline{u}_C$ . Similarly, if we increase  $\underline{u}_P$ , then  $\bar{\omega}$  strictly decreases because  $v(\theta_P, x^*(\theta_P)) > v(\theta_P, \hat{x})$ .
2. We assume without loss of generality that  $\theta_C < \theta_P$ . First we show that  $x^*(\theta_C) < \hat{x} < x^*(\theta_P)$ . Since  $\frac{\partial}{\partial x} v(\theta_P, x^*(\theta_P)) = 0$ , the single crossing property in Assumption 1 implies that  $\frac{\partial}{\partial x} v(\theta_C, x^*(\theta_P)) < 0$ . Thus, if the policy issue is contentious, i.e.,  $v(\theta_C, x^*(\theta_P)) < \underline{u}_C$ , then the strict concavity of utility in  $x$  implies that  $x^*(\theta_C) < \hat{x} < x^*(\theta_P)$ .

Because  $\hat{x} > x^*(\theta_C)$ , strict concavity of utility implies  $\frac{\partial}{\partial x}v(\theta_C, \hat{x}) < 0$ . Assumption 1 therefore implies that  $\frac{\partial}{\partial \theta}v(\theta_C, \hat{x}) > 0$ . As a consequence,  $\hat{x}'(\theta) = -\frac{\partial}{\partial \theta}v(\theta_C, \hat{x})/\frac{\partial}{\partial x}v(\theta_C, \hat{x}) > 0$ . Because  $\hat{x} < x^*(\theta_P)$  strictly concavity implies  $\frac{\partial}{\partial x}v(\theta_P, \hat{x}) > 0$ . Hence, raising  $\theta_C$  raises  $\hat{x}$  and therefore  $v(\theta_P, \hat{x})$ . Equation (18) therefore implies that  $\bar{\omega}$  and hence  $\bar{p}$  increases.

3. From (18), we have

$$\begin{aligned}\frac{d\bar{\omega}}{d\theta_P} &= \frac{\left(\frac{\partial}{\partial \theta}v(\theta_P, \hat{x})\right)(v(\theta_P, x^*(\theta_P)) - \underline{u}_P) - \left(\frac{d}{d\theta_P}v(\theta_P, x^*(\theta_P))\right)(v(\theta_P, \hat{x}) - \underline{u}_P)}{(v(\theta_P, x^*(\theta_P)) - \underline{u}_P)^2} \\ &= \frac{\left(\frac{\partial}{\partial \theta}v(\theta_P, \hat{x})\right)(v(\theta_P, x^*(\theta_P)) - \underline{u}_P) - \left(\frac{\partial}{\partial \theta}v(\theta_P, x^*(\theta_P))\right)(v(\theta_P, \hat{x}) - \underline{u}_P)}{(v(\theta_P, x^*(\theta_P)) - \underline{u}_P)^2},\end{aligned}$$

where the second equality is from the envelope condition. Since  $v(\theta_P, x^*(\theta_P)) > v(\theta_P, \hat{x})$ , and since the single-crossing property implies  $\frac{\partial}{\partial \theta}v(\theta_P, \hat{x}) < \frac{\partial}{\partial \theta}v(\theta_P, x^*(\theta_P)) \leq 0$ , we have  $\frac{d\bar{\omega}}{d\theta_P} < 0$ , proving the desired result.

■

**Proof of Proposition 3.** Suppose that  $p_1 > \bar{p}_2$ . If congress does not oppose the policy in the first period, then congress' expected payoff is

$$v_1(\theta_{C,1}, x^*(\theta_{P,1})) + \omega(p_1)v_2(\theta_{C,2}, x^*(\theta_{P,2})) + (1 - \omega(p_1))\underline{u}_{C,2}. \quad (19)$$

The possible benefit for congress of opposing the first-period policy, is that it lowers the updated prior  $p_2 = \pi_F(p_1)$  below  $\bar{p}_2$ , in which the president would not challenge congress in the second period. In this case, congress' ex-ante expected payoff from challenging the president is

$$\begin{aligned}\omega(p_1) &\left(v_1(\theta_{C,1}, x^*(\theta_{P,1})) + \omega(\pi_S(p_1))v_2(\theta_{C,2}, x^*(\theta_{P,2})) + (1 - \omega(\pi_S(p_1)))\underline{u}_{C,2}\right) \\ &+ (1 - \omega(p_1))\left(\underline{u}_{C,1} + v_2(\theta_{C,2}, \hat{x}_2^*)\right).\end{aligned} \quad (20)$$

Using the fact that  $v_2(\theta_{C,2}, \hat{x}_2^*) = \underline{u}_{C,2}$ , the difference between (20) and (19), i.e., congress' net-benefit of challenging the president in the first period is

$$\omega(p_1)(1 - \omega(\pi_S(p_1)))\left(\underline{u}_{C,2} - v_2(\theta_{C,2}, x^*(\theta_{P,2}))\right) - (1 - \omega(p_1))\left(v_1(\theta_{C,1}, x^*(\theta_{P,1})) - \underline{u}_{C,1}\right). \quad (21)$$

Let  $b_1 = v_1(\theta_{C,1}, x^*(\theta_{P,1})) - \underline{u}_{C,1}$  and  $b_2 = \underline{u}_{C,2} - v_2(\theta_{C,2}, x^*(\theta_{P,2}))$ . Because issue 1 is not contentious, while issue 2 is contentious, it follows that  $b_1, b_2 > 0$ . Thus, (21) implies that the net-benefit of challenging the president in the first period is strictly positive if

$$p_1\left((\omega_h - \omega_l)(b_1 + b_2) - (\omega_h^2 - \omega_l^2)b_2\right) > (1 - \omega_l)b_1 - (\omega_l - \omega_l^2)b_2. \quad (22)$$

By assumption  $\omega_l < 1$ . Thus

$$\text{sign}\left((1 - \omega_l)b_1 - (\omega_l - \omega_l^2)b_2\right) = \text{sign}(b_1 - \omega_l b_2). \quad (23)$$

Further,  $\omega_h > \omega_l$  implies that

$$\text{sign}\left((\omega_h - \omega_l)(b_1 + b_2) - (\omega_h^2 - \omega_l^2)b_2\right) = \text{sign}(b_1 + b_2(1 - \omega_l - \omega_h)). \quad (24)$$

Also note that the difference between the coefficient of  $p_1$  in (22) and the right-hand side of the equation is

$$(\omega_h - \omega_l)(b_1 + b_2) - (\omega_h^2 - \omega_l^2)b_2 - \left((1 - \omega_l)b_1 - (\omega_l - \omega_l^2)b_2\right) = (1 - \omega_h)(\omega_h b_2 - b_1). \quad (25)$$

We now analyze all in cases in which it is optimal for congress to oppose the president in the first period. First, we need that  $\pi_F(p_1) < \bar{p}_2$ , i.e., if the president was unsuccessful in the first period, then the updated prior is sufficiently low such that he will accommodate by proposing  $\hat{x}$  in the second period. Second, equation (22) must hold for  $p_1$ . We now check when the latter condition holds.

*Case 1:* Let  $\omega_l > b_1/b_2$ .

Thus,  $b_2\omega_l > b_1$  and (23) implies that the right-hand side of (22) is strictly negative.

If  $\omega_l + \omega_h \leq 1 + b_1/b_2$ , then (24) implies that the left-hand side of (22) is non-negative. Thus, (22) holds for all  $p_1$  with  $0 < p_1 < 1$ .

If  $\omega_l + \omega_h > 1 + b_1/b_2$  then (24) implies that the left-hand side of (22) is strictly negative. Because  $\omega_l > b_2/b_1$  we get  $b_2\omega_h > b_1$ . Thus, (25) implies that the coefficient of  $p_1$  in (22) is greater than the right-hand side (but still strictly negative). As a consequence (22) holds again for all  $p_1$  with  $0 < p_1 < 1$ .

*Case 2:* Let  $\omega_l = b_1/b_2$ .

Now (23) implies that the right-hand side of (22) is zero. Because  $\omega_h > b_1/b_2$  equation (24) implies that the coefficient of  $t$  on the left-hand side of (22) is strictly positive. Thus, (22) holds for all  $p_1 > 0$ .

*Case 3:* Let  $\omega_l < b_1/b_2$ .

Now (23) implies that the right-hand side of (22) is strictly positive. If  $\omega_h > b_1/b_2$  then (25) implies that the coefficient of  $p_1$  in (22) is greater than the right-hand side. Thus, there exists  $\bar{p}_1 < 1$  such that (25) holds for all  $p_1 > \bar{p}_1$ . Further, (25) implies that  $\bar{p}_1$  is given by (10).

If  $\omega_h \leq b_1/b_2$  then the coefficient of  $p_1$  in (22) is equal or strictly smaller than the right-hand side. Thus, (22) does not hold for  $p_1 < \bar{p}$ .

Next, if gridlock occurs, then the probability of gridlock in the first period is  $1 - \omega(p_1)$ , i.e., the probability that the president does not succeed. In order to get gridlock in the second period,

the president must be successful in the first period, which occurs with probability  $\omega(p_1)$ . In this case, the probability of gridlock is  $1 - \omega(\pi_S(p_1))$ . Thus, the overall incidence of gridlock in the second period is  $\omega(p_1)(1 - \omega(\pi_S(p_1))) = \omega_h(1 - \omega_h) + (1 - p)\omega_l(1 - \omega_l)$ .

Finally, Pareto inefficiency follows immediately. In particular, inefficiency in the second period follows from proposition 1. If there is gridlock then both players are strictly better off with policy  $x_p^*(\theta_p)$ . ■

**Proof of Proposition 4.** If  $\theta_{C,1}$  is moved away from  $\theta_{P,1}$  then  $\alpha$  as defined in (7) decreases, and  $\bar{\omega}$  stays the same. Thus,  $\bar{p}_2$  is unchanged. Thus, condition 1 (a) in Proposition 3 is still satisfied for  $p_1$ ,  $\omega_l$  and  $\omega_h$ . However, because  $\alpha$  is lowered, the condition becomes more slack and is satisfied by more parameter values.

Now suppose that we have case 1(b). Note that

$$\frac{\partial \bar{p}_1}{\partial \alpha} = \frac{(1 - \omega_h)(1 - \omega_l)}{(1 + \alpha - \omega_h - \omega_l)^2(\omega_h - \omega_l)} > 0.$$

Thus, lowering  $\alpha$  lowers  $\bar{p}_1$  which in turn means that the set of parameters for which gridlock occurs is at least weakly increasing.

Now suppose that  $\theta_{C,2}$  is moved away from  $\theta_{P,2}$ . Then  $v_2(\theta_{C,2}, x^*(\theta_{P,2}))$  again decreases, while  $\bar{\omega}_2$  and hence  $\bar{p}_2$  do not change. Hence, the argument is the same. ■

**Proof of Proposition 5.** Proposition 1 and the assumption that  $\omega_l < \bar{\omega}_2$  and  $\omega_h > \bar{\omega}_2$  implies that there exists  $0 < \bar{p}_2 < 1$ , such that the president offers  $\hat{x}_2$  in the second period, with  $\hat{x}_2 = \arg \max_x v_2(\theta_{P,2}, x)$  s.t.  $v_2(\theta_{C,2}, x) \geq \underline{u}_{C,2}$ . Congress approves of  $\hat{x}_2$  and hence there is no gridlock.

Suppose that  $p_1 < \bar{p}_2$  in the first period. If congress does not oppose the policy, no information is revealed, i.e.,  $p_2 = p_1$ . Therefore  $p_2 < \bar{p}_2$  and the president offers  $\hat{x}_2$ .

Now suppose that  $p_1 > \bar{p}_1$ , and congress opposes the policy that is offered. If the president can convince congress the policy, then the updated prior about the president's ability  $\pi_S(p_1) > \pi_S(\bar{p}_1) = \bar{p}_2$ , and hence the president offers policy  $x^*(\theta_{P,2})$  that solves  $\max_x v_2(\theta_{P,2}, x)$ .

Let  $p_1 \in (\bar{p}_1, \bar{p}_2)$ . If congress accepts policy proposal  $x$  in the first period, then its expected payoff is

$$v_1(\theta_{C,1}, x) + \underline{u}_{C,2}. \quad (26)$$

In contrast, if congress opposes  $x$ , then its expected payoff is

$$\begin{aligned} & \omega(p_1) \left( v_1(\theta_{C,1}, x) + \omega(\pi_S(p_1)) v_2(\theta_{C,2}, x^*(\theta_{P,2})) + (1 - \omega(\pi_S(p_1))) \underline{u}_{C,2} \right) \\ & + (1 - \omega(p_1)) \left( \underline{u}_{C,1} + v_2(\theta_{C,2}, \hat{x}_2) \right). \end{aligned} \quad (27)$$

Because  $v_2(\theta_{C,2}, \hat{x}_2) = \underline{u}_{C,2}$ , the cutoff policy  $\hat{x}_1$  for which congress is indifferent between opposing and not opposing, i.e., for which (26) equals (27), is given by

$$v_1(\theta_{C,1}, \hat{x}_1) = \underline{u}_{C,1} - \frac{\omega(p_1)\omega(\pi_S(p_1))}{1 - \omega(p_1)} (\underline{u}_{C,2} - v_2(x_2^*(\theta_{P,2}), \theta_{C,1})). \quad (28)$$

Note that  $v_2(x_2^*(\theta_{P,2}), \theta_{C,1}) < \underline{u}_{C,2}$ , because the second-period policy is contentious. Thus, if  $\tilde{u}_{C,1} = v_1(\theta_{C,1}, \hat{x}_1)$ , then  $\tilde{u}_{C,1} < \underline{u}_{C,1}$ . Further, it follows that congress is willing to accept any policy in the first period which gives a utility of at least  $\tilde{u}_{C,1}$ . This proves the first statement.

Now suppose that  $p_1 = \bar{p}_2$ . Then (28) implies that congress is strictly better off accepting  $x_1^*$  if and only if

$$\frac{\underline{u}_{C,1} - v_1(\theta_{C,1}, x_1^*(\theta_{P,2}))}{\underline{u}_{C,2} - v_2(\theta_{C,2}, x_2^*(\theta_{P,2}))} < \frac{\omega(\bar{p}_2)\omega(\pi_S(\bar{p}_2))}{1 - \omega(\bar{p}_2)} = \frac{(\omega_h - \bar{\omega}_2)(\bar{\omega}_2 - \omega_l) + \bar{\omega}_2^2}{1 - \bar{\omega}_2}. \quad (29)$$

Next, if  $x_1^*(\theta_{P,1})$  is in the interior of the policy space, then the president could possible force congress to oppose a policy by choosing  $x_2$  that makes congress even worse off. This is not the case if the president is better off if congress does not oppose policy  $x_1^*(\theta_{P,1})$ , i.e., if

$$\begin{aligned} v_1(\theta_{P,1}, x_1^*(\theta_{P,1})) + v_2(\theta_{P,2}, \hat{x}_2) &> (1 - \omega(\bar{p}_2))(\underline{u}_{P,1} + v_2(\theta_{P,2}, \hat{x}_2)) \\ &+ \omega(\bar{p}_2)(v_1(\theta_{P,1}, x_1^*(\theta_{P,1})) + \omega(\pi_S(\bar{p}_2))v_2(\theta_{P,2}, x_2^*(\theta_{P,2})) + (1 - \omega(\pi_S(\bar{p}_2)))\underline{u}_{P,2}), \end{aligned}$$

which is equivalent to

$$\begin{aligned} (1 - \omega(\bar{p}_2))(v_1(\theta_{P,1}, x_1^*(\theta_{P,1})) - \underline{u}_{P,1}) + \omega(\bar{p}_2)(v_2(\theta_{P,2}, \hat{x}_2) - \underline{u}_{P,2}) \\ > \omega(\bar{p}_2)\omega(\pi_S(\bar{p}_2))(v_2(\theta_{P,2}, x_2^*(\theta_{P,2})) - \underline{u}_{P,2}). \end{aligned} \quad (30)$$

Dividing by side of (30) by  $v_2(\theta_{P,2}, x_2^*(\theta_{P,2})) - \underline{u}_{P,2}$  and using the definition of  $\bar{\omega}_2$  it follows that (6) is equivalent to

$$\frac{v_1(\theta_{P,1}, x_1^*(\theta_{P,1})) - \underline{u}_{P,1}}{v_2(\theta_{P,2}, x_2^*(\theta_{P,2})) - \underline{u}_{P,2}} > \frac{\omega(\bar{p}_2)(\omega(\pi_S(\bar{p}_2)) - \bar{\omega}_2)}{1 - \omega(\bar{p}_2)} = \frac{(\omega_h - \bar{\omega}_2)(\bar{\omega}_2 - \omega_l)}{1 - \bar{\omega}_2}. \quad (31)$$

Thus, if (14) holds then congress is strictly better off accepting  $x_1^*$  and it is optimal for the president to offer this policy. Because the inequalities are strict, they also hold for some  $p_1 < \bar{p}_2$ .

■

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