Auction hosting site pricing and market equilibrium with endogenous bidder and seller participation☆

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Abstract

This paper characterizes the payoffs and pricing policies of auction hosting sites when both the bidders’ and the sellers’ participation is endogenous. Sellers have heterogeneous opportunity costs and make a listing decision depending on the listing fee and the expected revenue from the sale. On the other side of the market, factors such as facility in navigating an interface layout and prior bidding experience result in bidder heterogeneity with respect to participation costs. Bidders participate if their ex ante expected payoff from searching the site exceeds their participation costs. The auction site earns revenue by setting positive listing fees, trading off the increased revenue per seller resulting from a higher fee with the revenue reduction from the loss of sellers. Though this appears to be a classic monopoly problem, there are important differences. The reduction in the number of sellers participating in a site has feedback effects, as it affects the number of bidders who choose to visit that site, which in turn again affects the attractiveness of the site to sellers, and thus further reduces seller participation. In this environment the monopolist’s ability to extract rents is severely limited, even if one considers rent extraction from the seller side of the market only. It is demonstrated that the inverse demand curve is flatter than the demand curve obtained from the (inverse) distribution of seller costs. Moreover, the inverse demand curve has at least one and possibly multiple flat segments, leading to discontinuities in the profit function. Thus, small changes in the environment can lead to large changes in the optimal fee and market participation.

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1. Introduction

Auction hosting sites, whether of the brick-and-mortar or Internet variety, are intermediaries whose product is the provision of a marketplace in which buyers and sellers can transact. Their product is essentially a platform that connects two sides of a market rather than a traditional physical product. Much of the early literature on auctions abstracted from the presence of auction hosting sites and considered auctions to be an interaction between the owner of an item (the seller) and many potential buyers (the bidders). Moreover, the bulk of the recent auction literature that focuses on Internet auctions is primarily concerned with seller vs. buyer issues and the dynamics of prices (within a series of auctions or across time) rather than optimal host site policies (see Bajari and Hortacsu, 2004; Ashenfelter and Graddy, 2003, for two recent excellent surveys of this literature and associated issues).

Some of the recent literature recognizes that auctions are not merely interactions between a seller and a set of bidders. Rather, bidders have a choice of whether or not to attend the auction and, if so, for which possible seller’s item to compete. The first question is considered in the literature on bidder entry, while the second question is considered in the (much smaller) literature on competing sellers. Nearly absent is the recognition that sellers also have “outside” options (see Hernando-Veciana, 2005 and references for some relatively rare exceptions), and that sellers may not be competing with each other for a pool of potential bidders, but may be synergistic in creating a market place that fosters bidder participation.

This paper provides critical insights into filling the gap between the literature of endogenous entry and seller competition and the literature on platforms and intermediaries. Unlike the extant literature discussed above, we explicitly recognize the nature of the auction hosting site as a strategic intermediary, whose “product” is access of bidders to sellers and sellers to bidders. As a result, in contrast to the competing sellers literature, sellers need not be substitutes to each other; rather a site that attracts more sellers may be more valuable to sellers because it is a thicker market in terms of bidder participation. Our paper also explicitly recognizes heterogeneity among both sellers and buyers in terms of the value they attach to participating in the auction hosting site. Finally, we take into consideration the empirical regularity that auction hosting sites obtain revenue from sellers rather than from bidders. Unlike the literature on intermediaries and platforms which focuses on different market environments, we explicitly consider the structure of bidding competition and the associated extraction of revenue by the sellers.

We develop a model with the features described above and study the analytics of the equilibrium bidder entry and seller listing decisions, conditional on the auction site’s listing fees. We then derive the comparative statics of this entry equilibrium with respect to bidder heterogeneity, participation costs, and the auction site’s listing fee. This allows us to derive the auction site’s demand curve and its comparative statics. We show that this derived demand curve

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1 See Rochet and Tirole (2006), Armstrong (2006), Hagiu (2006), and references therein for a recent discussion on platforms and platform pricing, and Spulber (1999, 2006) for discussion on the broad literature on intermediaries.


4 Anderson et al. (2004) and Ellison et al. (2004), explicitly recognize the platform nature of auction hosting sites, but do not consider strategic behavior by the sites, or heterogeneity of seller/bidder preferences towards participation in them.
differs qualitatively from that of a standard monopolist. It is flatter than the demand curve obtained from the distribution of customer (in this case, seller) reservation values, and contains at least one perfectly flat segment even if there are no mass points in the distribution of customer willingness to pay. This shows that a monopolist auction hosting site has a severely limited ability to extract rent from its customers (the sellers). Moreover, the profit function is discontinuous and small changes in the environment or the listing fee can lead to large changes in site activity. In other words, in this environment, small changes in the underlying technology or preferences can lead to “transformational” changes in the overall market.

2. The model

We consider an auction hosting site whose sole source of revenue is generated by charging a listing fee, $f$, to sellers who wish to auction off their wares on the site. The auction hosting site incurs no costs of its own, but sellers and bidders incur costs of using the site. There is no direct interaction between the bidders and the host site. Thus, the auction hosting site is an intermediary who provides a (costly) platform, i.e., it makes the market in which potential buyers and sellers transact, and extracts a fixed amount of the seller’s surplus in the form of a listing fee.

2.1. Buyers

Consider a set of $N$ potential buyers who have an interest in purchasing a single unit of a particular type of item. A buyer anticipates that the item may be offered at auction at the auction hosting site. The maximum willingness of buyer $i \in \{1, \ldots, N\}$ to pay for a unit of the item, denoted by $v_i$, is an i.i.d. draw from the distribution $F(v)$ on $[0, \bar{v}]$ and is private information of each buyer. In order to ascertain this value, a buyer needs to obtain detailed information about the item. Thus, we assume that the bidder observes $v_i$ following his decision to visit the auction host site. Implicitly, we assume that there is information about the details of the item and this information is observed only upon visiting the site. Alternatively, the process of browsing the site and bidding is necessary for the potential buyer to cognitively identify the maximum willingness to pay for that item. The distribution of values, $F$, has the monotone hazard rate property and the functional form is common knowledge.

Entering the auction site and gathering information about the item(s) for auction is costly to the bidder. These costs include the costs of browsing the site to identify whether an item that he is interested in is indeed there, the costs of finding the item, reading its description, setting up an account (if he does not have one), bidding, and obtaining the item. Potential buyers differ in terms of their facility of navigating the layout and design of the auction host site, their prior experience with the site, and the speed of their internet connection. Therefore, the transaction costs differ from bidder to bidder. To capture this, we suppose that each bidder is characterized by a “location” $x_i$, distributed i.i.d. uniformly on the unit interval, which indexes his relative preference for the layout of the auction hosting site. Specifically, $x_i$ measures the distance in a preference space that the bidder has from the auction host site located at the origin.\(^6\) In addition to these

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5 The site cannot extract (directly) any surplus from the bidders since by assumption it does not charge a positive price for their access to the site.

6 The uniform distribution is used only for notational convenience as “location” maps to probability of participation. The nature of the analytics and the results are not affected by using a different distribution, except insofar as requiring an additional layer of notation and (possibly) precluding full bidder participation (as discussed below).
preference dependent costs, we assume that all bidders incur a fixed cost, \( c^B \), of transacting at the site. The total costs for bidder \( i \) of transacting at the host site are equal to \( t_i = c^B + \theta x_i \), where \( \theta \) captures the intensity of individual specific costs.\(^7\) We assume that a bidder’s location parameter \( x_i \) is private information, whereas the distribution of locations and the parameters \( c^B \) and \( \theta \) are common knowledge.\(^8\) A bidder’s outside option of not attending the host site yields a reservation utility of 0.

### 2.2. Sellers

Associated with an auction hosting site are a number of potential sellers, who each have a single item to offer for sale. Potential sellers have a cost of parting with their item that is equal to \( c^S \). Costs of parting with the item can be thought of as the transaction costs of putting it up for sale, such as the time and effort of establishing a presence on the site, the costs of preparing the item for sale, including providing a detailed description of the item, and uploading the information on the site, etc. We assume that these costs are private information and are i.i.d. draws from a commonly known distribution \( G(\cdot) \) with lower endpoint of 0. Sellers who choose not to list their item for sale receive a reservation utility of 0.\(^9\)

In this paper we abstract from competition between sellers. This is tantamount to assuming that potential sellers have items for which there is zero substitutability from the point of view of the buyers (e.g., rare items for which at most one will be on sale for any reasonably long period of time). Therefore, for expositional ease and without further loss of generality, we assume that it is common knowledge that there is only one potential seller who owns the item that the potential buyers are interested in. In this case market thickness is interpreted as the likelihood that the seller lists his item for sale at the site.

### 2.3. Sequence of events

The strategy space for each of the actors in this market is as follows. The site decides on the listing fee, \( f \), to charge to the sellers. This listing fee is public information (i.e., both the potential seller and potential bidders observe it). The potential seller privately observes his transaction cost, \( c^S \), and, given the listing fee \( f \), decides whether or not to pay the listing fee and put up his item for sale. Upon listing the item, the seller sets an optimal reserve \( \bar{v} \).

Buyers decide whether to incur the transaction cost \( t_i \) to visit the auction site in ignorance of the seller’s entry and reserve price decisions, but with knowledge of the listing fee, \( f \), and in anticipation of the reserve \( \bar{v} \) (i.e., in effect they move simultaneous to the sellers). If upon visiting the site a bidder identifies an item he is interested in purchasing, he observes his value \( v_i \) for the item. Bidding commences according to the English format, with the auction being concluded once no bidder desires to further raise the bid. If the final bid is greater than the reserve, this bidder obtains the item, otherwise no sale takes place. Payoffs of all players are realized at this time.

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\(^7\) This parametrization yields a bidder participation cost distribution that belongs to a two parameter location/dispersion family and allows in principle for easy comparative statics with respect to the mean and dispersion of these costs. More general cost structures do not qualitatively impact the remaining comparative statics derived in this paper.

\(^8\) Notice that this is similar to linear “transport” costs in a Hotelling (1929) type of model, albeit one that augments them with a cost component that is the same for all potential buyers and which contains only a single firm.

\(^9\) In principle, \( c^S \) can account for the seller’s own use-value of the item. However, under this second interpretation, \( c^S \) would potentially affect both the optimal reserve the seller would prefer to post and the reservation level of utility of not listing. These are unnecessary complications that do not affect the nature of our results.
3. Bidder and seller participation in the auction

In this section we examine the participation problem that the potential seller and potential bidders face. We subsequently characterize under what conditions trade occurs, for a given site listing fee, \( f \). We refer to constellations of bidder and seller participation in which the marginal seller and bidder type earns zero surplus,\(^{10}\) conditional on the listing fee, as the “entry equilibrium.” This is further developed in the subsequent section in which we analyze properties of the optimal listing fee and associated bidder/seller participation under the optimal fee, which we refer to as the “market equilibrium.”

3.1. The buyers’ attendance decisions

A potential bidder decides whether to incur his transaction cost \( t_i \) of attending and browsing the site in light of his expected benefit of being at the auction hosting site. Thus, he forms expectations of his payoff of being at the auction site.

To this end, suppose that a (as yet to be specified) reserve of \( v^\bar{\varphi} \) is in effect, then let the expected payoff of a bidder with valuation \( v_i \) from participating in an English auction in which there are \( m = n - 1 \) rivals be denoted by \( E\pi(v_i, n) \). Before his value \( v_i \) is revealed to him, but after he has incurred the transactions costs associated with going to and browsing the site, his expected payoff with \( m \) rivals in the same situation as himself (i.e., with a total of \( n = m + 1 \) participating bidders), conditioned on the item being available is:

\[
E\left[\pi(n)\right] := E_{v_i}E\pi(v_i, n) = \int_{v_i}^{\bar{v}} \int_{y}^{v_i} [F(y)]^{n-1}dy f(v_i)dv_i. \tag{1}
\]

Prior to going to the auction site, however, it is not yet known whether the seller who has the item that interests the bidder is actually listing at the auction site. Letting \( q \in [0, 1] \) denote the probability that the seller is at auction (given a listing fee of \( f \)), one can determine the threshold location of the critical bidder ex ante. That is, one can find the location, \( x^\tilde{\varphi} \), of the marginal bidder who is indifferent between browsing and not browsing the auction site, so that any bidder with \( x_i \leq x^\tilde{\varphi} \) attends the site, whereas those with \( x_i > x^\tilde{\varphi} \) refrain from attending the site.

In order to determine the critical bidder threshold, it should be noted that a bidder’s expected auction payoff is sensitive to how many rivals are at the auction. Thus, the bidder needs to evaluate the likelihood of facing different numbers of rivals at the auction.

Let \( x_i \) denote the location of bidder \( i \) in \([0, 1]\). And let \( x_i^{(i)} \) denote the location of the \( i \)th ordered bidder from the left on \([0, 1]\). Thus, \( x_i^{(1)} \) is the location of the bidder located most to the left (the smallest value of \( x_i \)) and \( x_i^{(N)} \) is the location of the bidder farthest to the right (i.e., the largest value of \( x_i \)). For arbitrary cut-off \( x' \in [0, 1] \), the probability that there are exactly \( n \in \{0, 1, \ldots, N\} \) potential bidders in the interval \([0, x'] \) is given by,

\[
Pr\{x_i^{(n)} \in [0, x'] \land x_i^{(n+1)} \not\in [0, x']\} = \binom{N}{n} x^n (1 - x')^{N-n}. \tag{2}
\]

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\(^{10}\) Except when all bidders participate, in which case positive bidder surplus is consistent with the entry equilibrium.
Hence,

**Lemma 1 (Critical Distance to Auction Site).** Given costs of entering the auction site $c^B + \theta x$ and positive bidder participation, bidder participation is characterized by a cut-off value for going to the auction $\tilde{x} \in (0, 1)$ that is implicitly defined by the following relationship:

$$q \sum_{m=1}^{N-1} \binom{N-1}{m} \tilde{x}^m (1-\tilde{x})^{N-1-m} E[\pi(m + 1)] = c^B + \theta \tilde{x}$$  \hspace{1cm} (3)

**Proof.** The critical bidder is just indifferent between visiting the site in the hope of finding and obtaining an item at the auction and staying out of the auction, obtaining a reservation utility of zero. Thus, the critical bidder’s expected payoff from going to the auction site must equal his expected costs of doing so. The right-hand-side of Eq. (3) gives the cost of attending the site, whereas the left-hand-side denotes the bidder’s *ex ante* expected payoff at the auction host site. This is given by the bidder’s expected payoff in an auction with a given number of bidders $n$ (as derived in Eq. (1)) weighted by the distribution of the expected number of bidders at the auction (as given in Eq. (2), adjusted for the fact that a participating bidder cares about the number of competing bidders). Finally, this expected payoff must be conditioned on the presence of the seller, a probability given by $q$. □

After re-writing the bidder participation condition in terms of the minimum necessary seller participation, $q$, that assures bidder participation up to $\tilde{x}$, we can analyze the properties of this relationship. To this end, we attach the superscript D to the seller participation as a mnemonic for demand for the object, signifying that the implied seller participation rate is that which is necessary to induce the bidder participation (i.e., expected demand for the object) given by the location threshold $\tilde{x}$:

$$q^D(\tilde{x}) = \frac{c^B + \theta \tilde{x}}{\sum_{m=1}^{N-1} \binom{N-1}{m} \tilde{x}^m (1-\tilde{x})^{N-1-m} E[\pi(m + 1)]}. \hspace{1cm} (4)$$

**Lemma 2.** The relationship between minimum seller participation and bidder attendance is increasing. That is,

$$\frac{d}{d\tilde{x}} q^D(\tilde{x}) > 0.$$

**Proof.** The numerator, i.e., the transaction costs of participating in the auction $t(\tilde{x})$, is linearly increasing in $\tilde{x}$. As for the denominator, an increase in $\tilde{x}$ leads to increases in the expected number of bidders $m + 1$ in the sense of first-order stochastic dominance.\textsuperscript{11} Since the bidder’s expected payoff as a function of the number of rivals $m$ is decreasing, the denominator is decreasing in $\tilde{x}$. Hence, $q^D(\tilde{x})$ is increasing in $\tilde{x}$. □

\textsuperscript{11} See Eq. (2) and, e.g., Wolfstetter (1999), p. 223.
Note that
\[
\left( \frac{N-1}{m} \right) \tilde{x}^m (1 - \tilde{x})^{N-1-m} \bigg|_{\tilde{x}=0} = \begin{cases} 1, & \text{for } m = 0 \\ 0, & \forall m > 0, \end{cases}
\]
and
\[
\left( \frac{N-1}{m} \right) \tilde{x}^m (1 - \tilde{x})^{N-1-m} \bigg|_{\tilde{x}=1} = \begin{cases} 0, & \forall m < N - 1 \\ 1, & \text{for } m = N - 1. \end{cases}
\]

Thus, Eq. (4) begins at \( q^D(0) = \frac{c^D}{E[\pi(1)]} > 0 \). Thereafter \( q^D(\tilde{x}) \) monotonically increases until it peaks at \( q^D(1) = \frac{c^D + \theta}{E[\pi(N)]} \). This relationship is depicted as the top line in Fig. 1 (in Figs. 1–7, thin lines denote \( q^D(\cdot) \)). Note that since bidder locations, \( x \), are uniformly distributed in [0, 1], \( \tilde{x} \) can be interpreted as the probability that a bidder participates in the auction, and thus both \( \tilde{x} \) and \( q \) are

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**Fig. 1. Market equilibrium with no transactions.**

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**Fig. 2. Market with a stable partial bidder participation equilibrium.**
probabilistic measures of market thickness. The region above this line identifies the combinations of seller and bidder participation probabilities under which bidders have positive expected payoffs; below the line their payoffs are negative.

3.2. The seller’s listing decision

Similarly to the analysis from the bidders’ perspectives, the potential seller will list at the site if and only if his expected payoff from doing so will cover his listing costs, made up of his idiosyncratic costs, $c^S$, and the auction listing fee, $f$. We first determine a seller’s expected payoff from being at the auction.
Suppose there are $n \in \{0, 1, \ldots, N\}$ bidders whose values are i.i.d. draws from the distribution $F(y)$ on $[0, \bar{y}]$. Then the expected revenue generated in an auction with (as yet arbitrary) reserve $\bar{y}$ is given by

$$E[R(n, \bar{y})] := n \int_{\bar{y}}^\infty \left( vf(v) - (1 - F(v)) \right) F(v)^{n-1} dv. \quad (5)$$

Thus, with the bidders’ cut-off value for going to the auction being denoted by $\tilde{x}$, the seller’s expected ex interim revenue from going to the auction, before knowing the number of bidders present, is given by

$$E[R(\tilde{x})] := E_n E[R(n, \bar{y})] = \sum_{n=0}^{N} \binom{N}{n} \tilde{x}^n (1 - \tilde{x})^{N-n} E[R(n, \bar{y})].$$

Fig. 5. Barely supported market.

Fig. 6. Comparative statics: changes in $\theta$, $c$, and $f$ that increase transaction volume (shifts are not necessarily parallel, and $\theta$ leads to a rotation around $q_0$).
This gives the seller’s expected revenue once he lists with the site, but before it is known how many bidders are present. Clearly, since bidders’ anticipated payoffs, given in Eq. (1), are a function of the optimally chosen reserve \( v^\ast \), the critical bidder participation cut-off, \( \tilde{x} \), given in Lemma 1, is a function of the optimally chosen reserve, i.e., \( \tilde{x} = \tilde{x}(v^\ast) \).

Recall, however, that the seller makes his entry and reserve decision independently and simultaneously to the entry decision of the bidders. An implication of this is that the seller cannot commit to an optimal reserve policy that would go against his best short-run interest in setting a reserve independent of the bidder participation decision. Hence, despite the fact that bidder participation is a function of the reserve chosen, the optimal reserve is that of a standard auction with deterministic bidder entry. That is, the optimal reserve is implicitly determined independently of the number of bidders by

\[
\nu = \frac{1 - F(v)}{f(v)}.
\]

Since \( F(\cdot) \) has the monotone hazard rate property, the optimal reserve is uniquely determined.\(^{12}\) Now, for given \( \tilde{x} \), a seller’s expected \textit{ex ante} payoff from going to the auction is

\[
E[R(\tilde{x})] - c^S - f. \tag{6}
\]

Letting the superscript \( S \) be a mnemonic for the analysis from the seller’s perspective (i.e., the supply side of the market), we state without proof:

**Lemma 3 (Seller Participation).** For a distribution of sellers’ costs \( G(c^S) \) and \( E[R(\tilde{x})] > f \), the probability that a seller attends the site is given by:

\[
q^S(\tilde{x}) = Pr\{c^S \leq E[R(\tilde{x})] - f\} = G\left( \sum_{n=1}^{N} \binom{N}{n} \tilde{x}^n (1 - \tilde{x})^{N-n} E[R(n, v)] - f \right); \tag{7}
\]

otherwise \( q^S = 0 \).

\(^{12}\) See Wolfstetter (1999) pg. 213. If the seller can establish a reputation or can commit in advance to a reserve, a lower reserve is chosen in order to increase bidder participation (see Levin and Smith, 1994; Wolfstetter, 1999).
An analysis of \( q^S(\hat{x}) \) yields:

**Lemma 4.** Seller participation \( q^S(\hat{x}) \) is weakly increasing in bidder attendance. That is,

\[
\frac{d}{d\hat{x}} q^S(\hat{x}) \geq 0.
\]

**Proof.** Define

\[
\hat{x} := \max\{\hat{x} | q^S(\hat{x}) = 0\}.
\]

Then \( \forall f > 0, \hat{x} > 0 \) and \( \hat{x} \) is the threshold for bidder participation that at a minimum must be assured to make any seller participation worthwhile. As noted in the proof to Lemma 2, an increase in \( \hat{x} \) leads to increases in the expected number of bidders \( N \) in the sense of first-order stochastic dominance and since the seller’s expected revenue is increasing in the number of bidders, it follows that \( q^S(\hat{x}) \) is increasing. \( \square \)

Thus, the behavior of Eq. (7) is such that it is constant at \( q^S = 0 \) for \( 0 < \hat{x} \leq \hat{x}_0 \). Thereafter, for \( \hat{x} \geq \hat{x}_0 \), it monotonically increases, peaking at \( q^S(1) = G(E[R(N, \hat{x})] - f) \). This relationship is depicted in Fig. 1 (in Figs. 1–7, thick lines denote \( q^S(\cdot) \); note that neither \( q^D(\cdot) \) nor \( q^S(\cdot) \) are defined to the right of \( \hat{x}_1 = 1 \). The region below the lower line identifies the combinations of seller and bidder participation under which the seller has a positive expected payoff; otherwise his payoff is negative.

**4. Entry equilibrium for given \( f \)**

Before conducting an equilibrium analysis of the optimal listing fee, we first consider potential equilibrium configurations, given an exogenously given listing fee \( f \). We refer to this as the “entry equilibrium,” in contrast to the “market equilibrium” that emerges as the solution to the auction hosting site’s pricing problem, given behavior in the entry equilibrium. The entry equilibrium is implied by the bidders’ and the seller’s participation problems given by Eqs. (4) and (7). We denote the seller participation rate in the entry equilibrium by \( q^* \) and that of bidders by \( \hat{x}^* \).

Thence,

**Lemma 5 (Existence of Entry Equilibrium).** An entry equilibrium for given \( f \) exists for all \( \hat{x}^* \in [0, 1] \) such that

\[
q^* = q^S(\hat{x}^* | f) \geq q^D(\hat{x}^*) \geq 0,
\]

with \( q^S > q^D \) only if \( \hat{x}^* = 1 \).

**Proof.** By Lemma 2, for given \( \hat{x} \), \( q^D \) determines the minimum seller participation necessary to induce bidders to the left of \( \hat{x} \) to participate. Thus, for given \( \hat{x} \), in equilibrium, \( q^S \geq q^D \), with inequality only if \( \hat{x}^* = 1 \). \( \square \)

**Fig. 2** gives a representation of the bidder and seller participation decisions and, thus provides a good illustration of Lemma 5.

Recall from **Fig. 1** that the area below \( q^S \) gives all combinations of \( \hat{x} \) and \( q \) for which sellers make positive profits. The area above \( q^D \) is the region in which bidders make positive profits. On
the two lines, the “marginal” trader (i.e., the marginal seller for \( q^S \) and the marginal bidder on \( q^D \)) make zero profit — although infra-marginal traders earn positive payoffs. A market fails to exist in the region between the two curves as neither bidders nor sellers have non-negative payoffs. Equilibrium configurations occur whenever the curves intersect.

Notice that it trivially follows from Lemma 5 that there is always at least one equilibrium constellation:

**Definition 1 (No-Trade Equilibrium).** In a no-trade equilibrium (NTE) the auction hosting site is vacant, so there are no sellers and no bidders, i.e., \( q^* = \tilde{x}^* = 0 \).

The NTE — unless it happens to be the unique equilibrium — is a coordination failure equilibrium. Our focus is on equilibrium configurations in which trade occurs with positive probability.

**Definition 2 (Entry Equilibrium with Trade).** An entry equilibrium with trade (EWT) is an entry equilibrium with positive expected trade. We have \( 0 < \tilde{x}^* \leq 1 \) and \( 0 < q^* < 1 \).

- If \( \tilde{x}^* = 1 \), we refer to an EWT with full (bidder) participation (FPE).
- If \( \tilde{x}^* < 1 \), we have an EWT with partial (bidder) participation (PPE).

Both the NTE as well as two EWT are identified in Figs. 2 and 3 (in Fig. 3, one EWT is an FPE). Note that an entry equilibrium with trade only implies a positive probability of trade, not a probability of trade with probability 1: it may be than no bidder has a cost draw low enough to participate in the auction, or that there is no item for sale even if one is desired by some prospective bidders. Simply put, not everyone who shops in eBay finds something they like, and not every prospective seller is able to complete a sale. The existence of the FPE is characterized in the following proposition.

**Proposition 1 (Existence of FPE).** A necessary and sufficient condition for the existence of a FPE is that

\[
q^D(1|N) = \frac{e^B}{E[\pi(N)]} \leq G(E[R(N, \nu)] - f) = q^S(1|f, N).
\]

**Proof.** The proof follows trivially from the equilibrium condition given in Lemma 5. □

Of course, Proposition 1 is not necessary for the existence of other (non-FPE) entry equilibrium configurations with trade. Indeed, there may be multiple equilibrium points, as is indicated in Fig. 4. Moreover, if the distribution of bidder “locations” is not bounded above but rather has unbounded support, FPE is not possible, but the analysis of all other equilibria would not be affected.

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13 Note that if the support of seller participation costs is unbounded, i.e., \( G(e^S) < 1 \), \( \forall e^S < \infty \), then \( q^* = 1 \) can be ruled out. Moreover, however, even if \( e^S \) is bounded above, \( q^* = 1 \) can only emerge as a limiting case, since otherwise the host site could increase the listing fee without affecting seller participation (thus, also without affecting bidder participation), which results in strictly greater profit for the host site. We therefore suppress this case in the analysis of the entry equilibrium.

14 Moreover, if the distribution of bidder “locations” is not bounded above but rather has unbounded support, FPE is not possible, but the analysis of all other equilibria would not be affected.
Corollary 1 (Multiple EWT). If the inequality given in Proposition 1 is strict, then there exist three types of equilibrium configurations:

- an NTE (i.e., \( x^* = 0 \)),
- one (or multiple) PPE (for which, \( 0 < x^* < 1 \)),
- a FPE (i.e., \( x^* = 1 \)).

Proof. The proof follows since \( q^D(0) > q^S(0|f) \), and both \( q^D \) and \( q^S \) are continuous functions, so, given the condition in Proposition 1, there exists at least one \( x^* \in (0, 1) \) such that \( q^D(x^*) = q^S(x^*|f) \). □

Note that all three configurations are depicted in Figs. 3 and 4. In addition to these type of equilibrium configurations a further configuration that plays a critical role in our subsequent analysis of the market equilibrium may obtain.

Definition 3 (Knife-Edge Equilibrium). If, rather than crossing at the entry equilibrium, \( q^D \) and \( q^S \) only have a point of tangency in common, we refer to this entry equilibrium as a knife-edge equilibrium, KEE.

A KEE is depicted in Fig. 5. For this particular KEE a small increase of participation costs on either side of the market leads to a complete collapse of the market onto the NTE. Thus, for the services of the host site to be utilized a minimum of bidder and seller participation must be assured in expectation. Given this minimum participation, both sides of the market (sellers and buyers) have strictly positive expected surplus. However, even a minor decrease in participation from that minimum level leads to a collapse that destroys all surplus (the switch from the KEE to the NTE).

The possibility of multiple equilibrium configurations of the entry game calls for a qualitative understanding of these. Indeed, despite the interrelationship between bidder and seller decisions, the entry equilibrium configurations can be characterized by otherwise standard stability and efficiency criteria. Bidders’ actions are strategic substitutes for one another, that is, a bidder is less likely to want to attend the site, the more rivals intend to go to the auction. Nevertheless, since \( q^D \) is increasing, the collective individual best responses of the buyers can be viewed as a strategic complement to the seller participation rate. Moreover, as \( q^S \) is also increasing, the bidder participation threshold, \( \hat{x} \) and the seller participation rate \( q \) are strategic complements in that the more likely it is that a seller will be present, the more worthwhile it is for a bidder to show up and vice versa. In other words, in a (somewhat loose) interpretation of \( q^D \) and \( q^S \) as best-reply functions (in which \( q^D \) is understood to be the collective result of individual bidder optimization), we are faced with the case of upward sloping reaction curves and strategic complements. This allows for an analysis that is otherwise standard.

The notion of stability that we evoke here is an application of trembling hand perfection, namely the standard one in which the equilibrium is considered stable whenever minor deviations lead back to the equilibrium if there were a dynamic adjustment process. For interior equilibrium configurations this implies that \( q^S \) intersects \( q^D \) from above so that \( \left( \frac{d}{df} \right) q^S(\hat{x}^*|f) > \left( \frac{d}{df} \right) q^D(\hat{x}^*) \). More generally, the NTE and the FPE are both always stable; whereas the derivative condition must be met for a PPE to establish stability. A special case is the knife-edge equilibrium, KEE, in that it is ‘stable’ from above, but not below.
In addition to stability properties, the equilibrium configurations can also be ranked by efficiency:

**Theorem 2 (Maximal Entry Equilibrium).** Let \( \{\bar{x}_k^*\} \) for \( k = 1, \ldots, K \) denote the set of critical location thresholds associated with all entry equilibrium configurations. Define

\[
\bar{X}^* := \max \{\bar{x}_k^*\}.
\]

Then \( \bar{X}^* \) is associated with the Pareto-optimal entry equilibrium and this entry equilibrium is stable or knife-edge (i.e., upward stable).

**Proof.** Higher values of \( \bar{x}_k^* \) are associated with higher \textit{ex ante} bidder surplus, as \textit{ex ante} surplus of bidder \( i \) with location \( x_i \) is given by \( \max\{0, \theta(\bar{x}_k^* - x_i)\} \) for all \( x_i < 1 \) (the FPE yields even higher bidder surplus as the surplus of all potential bidders is positive). By the monotonicity of the \( q^D(\cdot) \) and \( q^S(\cdot) \) functions, higher values of \( \bar{x}_k^* \) are associated with higher corresponding equilibrium values of seller participation probabilities, \( q_k^* \). The \textit{ex ante} payoff of a seller with a cost \( c_S \) is given by \( \max\{0, G^S(q_k^*) - c_S\} \). This establishes that the entry equilibrium associated with \( \bar{x}_k^* \) is Pareto optimal. Stability of the entry equilibrium associated with \( \bar{x}_k^* \) follows directly from Proposition 1 for the case of a FPE (i.e., \( \bar{X}^* = 1 \)); and otherwise (if \( q^D(1) > q^S(1) \)) strict monotonicity and continuity of \( q^D \) and \( q^S \) imply that \( \left( \frac{d}{df} \right) q^S(\bar{X}^*|f) \leq \left( \frac{d}{df} \right) q^D(\bar{X}^*) \), where this expression holds with an equality at the knife-edge equilibrium. \( \square \)

Ruling out complete coordination failure (i.e, the NTE), we further assume that market participants coordinate on the stable equilibrium with positive market participation and a high probability of trade occurring. That is, bidders and sellers coordinate on entry equilibrium configurations with “thick markets,” characterized by \( \bar{X}^* \), given in Theorem 2.15

For this equilibrium it is important to consider how it is affected by changes in the model. The stability property of this equilibrium has some critical implications. In particular we have:

**Lemma 6.** Decreases in transactions costs on either side of the market lead to an overall increase in market participation on both sides of the market. That is,

\[
\frac{d\bar{x}^*}{df} \leq 0, \quad \frac{dq^*}{df} \leq 0; \quad \frac{d\bar{x}^*}{dc^B} \leq 0, \quad \frac{dq^*}{dc^B} \leq 0; \quad \frac{d\bar{x}^*}{d\theta} \leq 0, \quad \frac{dq^*}{d\theta} \leq 0,
\]

with strict inequalities on all derivatives in a PPE, and in the first derivative in a PPE and FPE (the derivatives are not defined in a KEE).

**Proof.** Consider first a FPE so that \( \bar{X}^* = 1 \). Then, \( q^* = q^S(1) = G(E[R(N, y)] - f) \) and the proof follows readily. Consider next a PPE, so that 0 < \( \bar{X}^* < 1 \). Then, by Lemma 5, in equilibrium, Eqs. (4) and (7) must simultaneously hold. Differentiating both with respect to \( f \) yields

\[
\frac{\partial q^D}{\partial \bar{x}} \frac{d\bar{x}^*}{df} - \frac{dq^*}{df} = 0,
\]

\[
\frac{\partial q^S}{\partial \bar{x}} \frac{d\bar{x}^*}{df} - \frac{dq^*}{df} + \frac{\partial q^S}{\partial f} = 0.
\]

\[\text{Indeed, as shown below, it is in the interest of the hosting site to maximize participation, and the auction site could in principle employ simple surplus reallocation strategies (e.g., participation bonuses) to render this configuration as the unique equilibrium configuration.}\]
Solving the first equation for \( d\hat{x} \) and substituting into the second equation we get

\[
\frac{\partial q^S}{\partial \hat{x}} \frac{dq^S}{df} \frac{d\hat{x}}{df} - \frac{dq^S}{df} + \frac{\partial q^S}{\partial \hat{x}} = 0.
\]

Multiplying through with \( \frac{\partial q^D}{\partial \hat{x}} \) we obtain

\[
\frac{\partial q^S}{\partial \hat{x}} \frac{dq^S}{df} \frac{d\hat{x}}{df} - \frac{dq^S}{df} \frac{\partial q^D}{\partial \hat{x}} + \frac{\partial q^S}{\partial \hat{x}} \frac{\partial q^D}{\partial \hat{x}} = 0.
\]

Solving for \( dq^S / df \) yields

\[
\frac{dq^S}{df} = -\frac{\partial q^S}{\partial \hat{x}} \frac{\partial q^D}{\partial \hat{x}} - \frac{\partial q^S}{\partial \hat{x}} \frac{\partial q^D}{\partial \hat{x}} = -\hat{x} \frac{\partial q^D}{\partial \hat{x}} \frac{\partial q^D}{\partial \hat{x}},
\]

which is negative by Lemma 2 and the stability condition of the equilibrium as given in the proof of Theorem 2. Observe, next, that a change in the listing fee affects \( \hat{x} \) only through a shift in \( q^S(\cdot) \) and not through a shift in \( q^D(\cdot) \). Given that \( q^D(\cdot) \) is increasing in \( \hat{x} \) for \( \hat{x} < 1 \) and bidder surplus is strictly positive \( \hat{x} = 1 \), \( \frac{d\hat{x}}{df} < 0 \) implies that \( \frac{d\hat{x}}{df} < 0 \) for \( \hat{x} < 1 \) and that \( \frac{d\hat{x}}{df} = 0 \) for \( \hat{x} = 1 \). The remaining results follow from similar reasoning. □

The result is intuitive and a reflection of the stability of the entry equilibrium (the converse of the Lemma is true in the comparative static for a non-stable equilibrium configuration). Fig. 6 gives an illustration of the Lemma. A slightly less straightforward case is given by changes in the number of potential bidders, \( N \), as given by the next Lemma.

**Lemma 7.** Increases in the number of potential buyers leads to an increase in seller participation in equilibrium.

This lemma is illustrated in Fig. 7. An increase in \( N \) has the effect to increase both \( q^D \) and \( q^S \). The shift in \( q^S \) is straightforward since more bidders raise revenue. The shift in \( q^D \) is straightforward due to increased competition from rival bidders. As a result of the latter, the equilibrium bidder participation threshold may shift in either direction, despite the overall attendance (in terms of number of expected bidders) at the auction being increased. Seller participation, though, increases. To see this, suppose bidder locations are known. Then clearly, holding \( q \) constant, an increase in \( N \) by one bidder will either leave the number of participating bidders, \( n \), unchanged or increase it by one. It will leave it unchanged if the additional bidder has location that is further out than the \( x(n) \) by so much that his profit of competing against \( n \) rival bidders is negative, or if he falls to the “left” of \( x(n) \), but the previously marginal participating bidder earns negative profit competing against \( n \) rival bidders (and will thus exit). It will increase it by 1 if the additional bidder has location that is closer than \( x(n) \) but that the previously marginal participating bidder still earns positive profit when competing against \( n \) other bidders. Integrating over the distribution of the possible locations of the bidders should provide the result that the number of participating bidders first order stochastically increases with \( N \) holding \( q \) constant, and
thus the equilibrium value of $q$ increases with $N$.\footnote{Another way to approach this is to observe that a sufficient condition for $N$ to increase $q$ is if for $\theta=0$ (the Levin and Smith, 1994 case) an increase in $N$ increases seller surplus. This is guaranteed to happen for relatively small values of $N$ and when bidder entry is in pure strategies (as in our model).} We now turn to the market equilibrium, given by the solution to the auction hosting site’s problem of choosing a listing fee $f$.

5. Market equilibrium and choice of $f$

We now consider the market equilibrium of the auction hosting site, assuming throughout that the entry equilibrium is characterized by $X^\ast$. The auction site maximizes its revenue by choosing the listing fee, $f$, taking into account how this affects the (maximal) entry equilibrium, i.e., accounting for how the site fee affects the probability of sellers coming to auction, and thus affects bidder participation. In order to make sure that equilibrium configurations that involve potential trade exist, we make the following assumption.

**Assumption 1 (Potential for Positive Trade).** When the auction host site operates free of any listing fee to the seller ($f=0$) then there is potential for trade. That is,

$$\exists \hat{x}'>0 \text{ s.t. } q^S(\hat{x}'|f=0) \geq q^D(\hat{x}') .$$

The assumption essentially requires participation costs to be sufficiently low and the density of $G(\cdot)$ at zero to be sufficiently high: Since for $c^B=0$ and $f=0$ both $q^D(\cdot)$ and $q^S(\cdot)$ start at the origin, if $q^S(\cdot)$ is steeper than $q^D(\cdot)$ (i.e., if the density of $G(\cdot)$ at the origin is sufficiently high) Assumption 1 is satisfied (and by continuity will also be satisfied for low enough participation costs).

We now consider how changes in the host site’s listing fee affect the equilibrium with trade. Specifically, changes in $f$ affect the location of $q^S$, while leaving $q^D$ unaffected. Increases in $f$ lead to a downward shifting of $q^S$, although this shift is not uniform, as it depends on the distribution of seller costs, $G$ (unless $G$ itself is uniform). Given the entry equilibrium, we obtain,

**Proposition 3.** Seller participation in the entry equilibrium, $q^\ast$, is decreasing in the listing fee $f$, but not everywhere differentiable. Consequently, $\Delta f>0 \Rightarrow \Delta q^\ast \leq 0$ with strict inequality whenever $q^\ast>0$.

**Proof.** Suppose that $q^\ast$ is differentiable in $f$. Then, the result is given by Lemma 6. Suppose next that $q^\ast$ is not differentiable in $f$. Then an decrease in $f$ either leads to a jump in the entry equilibrium to a knife-edge equilibrium, which (by Theorem 2) entails lower participation, or a FPE is reached, with properties given above (conversely, an increase in $f$ from a knife-edge equilibrium results in a jump to a lower EWT equilibrium or a NTE. There must be at least one point of discontinuity in $q^\ast$ as a function of $f$ when $f>0$ since $q^S(\hat{x})=0$ for sufficiently small values of $\hat{x}$ when $f>0$ (because $E[R(\hat{x})]-f<0$ for sufficiently small $\hat{x}$) and $q^D(\hat{x})$ is increasing in $\hat{x}$ (and $q^S$ has a non-negative $x$-axis intercept). Thus, for sufficiently high $f$, $q^\ast$ cannot smoothly decrease to zero, but must jump to the no-trade equilibrium. \(\square\)

Proposition 3 establishes, in a sense, a downward sloping demand function for the host site’s services. That is, it generates the relationship between equilibrium seller participation and the host site’s listing fee. Unfortunately, this demand function is generally not well-behaved; that is, it has
both non-differentiable points (kinks) and infinitely elastic portions (jumps). In particular, for very high site listing fees, \( q^S \) will be so small that \( q^D > q^S \) for all \( \bar{x} > 0 \), and the NTE is the unique equilibrium (see Figs. 1, 8–12). Decreases in \( f \) shift \( q^S \) upward until ultimately a knife-edge entry equilibrium is first obtained (see Fig. 5). This equilibrium becomes stable as \( f \), and thus \( q^S \) is further increased (Fig. 2). Given such further decreases in \( f\), either the equilibrium continues to shift upwards, or a new knife-edge equilibrium with higher participation becomes feasible (e.g., the jump from \( \tilde{q} \) to \( \bar{q} \) in Fig. 12). Ultimately, no additional equilibrium configurations with discretely higher participation become available, and the equilibrium may, but need not, converge into a FPE (see \( q^{\text{FPE}} \) in Figs. 8–12).

For listing fees for which a FPE exists, further increases in \( q^* \) as \( f \) falls are a direct reflection of the distribution of the seller’s participation cost, i.e., the host site’s demand curve is given by the equation \( E[R(N, v)] - G^{-1}(q_{eq}) \) for low enough \( f \) (see Fig. 8). In this region the host site’s problem is a standard monopoly problem. However, short of full bidder participation, demand becomes more elastic, which severely restricts the rent extraction abilities of the host site, as the entry equilibrium is subjected to negative feedback effects from seller participation to bidder participation and back again.

**Fig. 8.** Auction site demand and distribution of seller participation costs.

**Fig. 9.** Optimal auction house pricing: stable market equilibrium.
Since short of the FPE the demand is a function of the interaction of bidder and seller participation, decreases in transactions costs on the bidder side augment demand. We have the following corollary to Lemma 6:

**Lemma 8.** In any PPE, reductions in bidder transactions costs leads to increases in the host site’s demand.

*Fig. 11* illustrates this effect. There it is also shown that the kinks in the demand curve are shifted inwards, exposing less elastic demand in larger ranges. Due to the behavior of demand for the services of the host site, the auction hosting site’s revenue (i.e., profit) function has irregularities (see *Fig. 13*). Thus, the profit function has the potential for a non-differentiable point (a kink). This happens where seller participation drops below the point which assures an FPE (given at the fee $f^{\text{FPE}}$ in *Figs. 12 and 13*). In other words, for small $f$, the implied entry equilibrium is the one in which $\tilde{X}^* = 1$, for larger $f$ (and, thus, decreased $q^S$) it entails $\tilde{X}^* < 1$. The host site’s objective function is continuous in $f$, but not differentiable at the point where one switches from the first to the second type of equilibrium.

*Fig. 11* Demand comparative statics: decrease in $\theta$ and $c$. 

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**References:**

In addition to the kink, the host site’s profit function also has discontinuities (i.e., jumps). Jumps are always downward and happen at the fee levels that correspond to the flat portions of the inverse demand, i.e., when a marginal change in the listing fee results in a jump to an entry equilibrium with discretely lower participation levels (see the jump from $\hat{q}(\hat{f})^{\text{PPE}}$ to $\bar{q}(\hat{f})^{\text{PPE}}$ in Fig. 12 that is associated with $\hat{f}$ in Fig. 13).

Analytically, the host site’s objective can be broken into two parts. Letting $\bar{f}^{\text{FPE}}$ denote the level of $f$ at which one switches from a FPE to a PPE, the host site’s objective function is

$$
q_S(1|f) \times f = G(E[R(N,v)] - f) \times f \text{ if } f < \bar{f}^{\text{FPE}},
$$

$$
q_S(\tilde{X}^*|f) \times f \text{ if } f \geq \bar{f}^{\text{FPE}},
$$

where the strict inequality limiting the domain of the first branch and the weak inequality on the second branch stem from the fact that as one switches from an FPE (at a fee of $\bar{f}^{\text{FPE}}$) to a PPE the elasticity of seller participation with respect to the listing fee $f$ drops.

Fig. 12. Generalized auction hosting site demand with multiple interior market equilibria.

Fig. 13. Profit function with multiple interior market equilibria.
The market equilibrium can occur on any portion of the profit function, including where there are kinks or jumps. The formal conditions for these equilibrium configurations are summarized thus:

**Theorem 4 (Market Equilibrium).** The following market equilibrium constellations can occur, where \( f^{**} \) denotes the optimal listing fee and \( q^{**} \) and \( x^{**} \) the associated entry equilibrium values of seller and bidder participation.

- **A FPE where the profit function is not at a kink:**

\[
0 < f^{**} = \frac{G(E[R(N, v)] - f^{**})}{g(E[R(N, v)] - f^{**})} < f^{\text{FPE}}
\]

with \( f^{\text{FPE}} = E[R(N, v)] - G^{-1}\left(\frac{\theta B}{E[\pi(N)]}\right)\).

- **A FPE at the kink:**

\[
f^{**} = f^{\text{FPE}} \quad \text{and} \quad \hat{x}^{**} = 1,
\]

with
\[
\left[\frac{\partial q^{S}}{\partial f} + \frac{\partial q^{S}}{\partial \hat{x}} \frac{d\hat{x}^{*}}{df}\right] f^{\text{FPE}} + q^{S}(1, f^{\text{FPE}}) \leq 0
\]

and
\[
-g(E[R(N, v)] - f^{\text{FPE}}) \times f^{\text{FPE}} + q^{S}(1, f^{\text{FPE}}) \geq 0
\]

- **A PPE where the profit function is not at a jump:**

\[
f^{**} > f^{\text{FPE}} \quad \text{and} \quad \hat{x}^{**} < 1,
\]

with
\[
\left[\frac{\partial q^{S}}{\partial f} + \frac{\partial q^{S}}{\partial \hat{x}} \frac{d\hat{x}^{*}}{df}\right] f^{**} + q^{S}(\hat{x}^{**}, f^{**}) = 0
\]

- **A KEE (i.e., a PPE at a jump):**

\[
\frac{d}{d\hat{x}} q^{S}(\hat{x}^{**}) = \frac{d}{d\hat{x}} q^{D}(\hat{x}^{**}),
\]

with
\[
\left[\frac{\partial q^{S}}{\partial f} + \frac{\partial q^{S}}{\partial \hat{x}} \frac{d\hat{x}^{*}}{df}\right] f^{**} + q^{S}(\hat{x}^{**}, f^{**}) \geq 0
\]

Examples of the third and fourth equilibrium constellations are illustrated in Figs. 9 and 10.

The multiplicity of possible equilibrium configurations is a result of the fact that the auction hosting site’s marginal revenue curve jumps down at the kink and up at the points of discontinuity.

At points of discontinuity, small changes in the environment (e.g., changes in transaction costs, distribution of valuations, etc.) can have dramatic changes in the profit maximizing fee (and equilibrium participation) when the optimal fee moves across these points of discontinuity. In the knife-edge equilibrium, the functions \( q^{D} \) and \( q^{S} \) are tangent to one another, so a further increase in \( f \) leads to a (partial) market collapse (see Figs. 5, 9 and 13), possibly resulting in a NTE. In this case the first order condition for profit maximization of the host site need not be satisfied.
Conversely, small changes in the environment (e.g., changes in transaction costs, distribution of valuations, etc.) may have no change in the optimal fee whenever the optimal fee corresponds to the kink, i.e., whenever the entry equilibrium is at the cusp between being a PPE and a FPE. Here, too, the first order conditions of the objective function need not be satisfied.

It follows from the fact that demand is increasing in market participation (see Lemmas 6 and 8) that the market equilibrium configurations listed first in Theorem 4 are more likely to occur when bidders’ values are relatively high and transactions costs small. Conversely, if valuations are relatively small and transactions costs high, the latter market equilibrium configurations of Theorem 4 are more likely to occur. However, if the distribution of bidder “locations” is not uniformly distributed in the interval [0, 1] but rather has unbounded support, then only the latter two types of equilibrium configurations are possible.

6. Conclusion

The preceding analysis was conducted in two steps. The first step focuses on the existence and properties of an entry equilibrium in which bidders and sellers make use of the auction hosting site. The second step derives the auction site’s demand curve implied by the entry equilibrium configurations, under the assumption that the host site charges sellers listing fees and sellers and bidders attain the highest activity entry equilibrium, given the listing fee.

The most important insights from the analysis of the entry equilibrium are the following. First, the entry equilibrium is characterized by subtle mutual feedback effects from bidder participation to seller participation and vice versa. As a result, while bidders’ actions are strategic substitutes to each other (the more bidders attend the site, the less lucrative it is to attend), the aggregate effect of their actions are strategically complementary to the sellers’ participation decision — leading to some results that are similar to those found in macro-economic coordination games, despite the fact that individual bidders’ entry decisions are not strategically complementary vis-a-vis their potential rivals’ decisions.

Second, conditional on the existence of entry equilibrium configurations, there are generally multiple configurations. These can be ranked by efficiency in increasing order of participation. Moreover, the most efficient entry equilibrium is also stable; unless it is a ‘knife-edge’ equilibrium, the implications of which we discuss in the next point.

Third, due to the subtle mutual feedback effects from the two sides of the market, small changes in the environment may lead to dramatic changes in the entry equilibrium. Indeed, the market may collapse, or make discrete jumps to and from different levels of market activity whenever the entry equilibrium is a knife-edge equilibrium. In particular, jumps can happen when a small increase in the host site’s listing fee results in the elimination of the hitherto highest activity equilibrium and thus forces a transition to another equilibrium that corresponds to a discretely lower activity level.

As a result of the behavior of the entry equilibrium, the demand for the host site’s services are not generally well-behaved even under the assumption of full coordination in the market. In particular, the (inverse) market demand has at least one and potentially multiple flat segments that correspond to discrete changes in seller participation as the market equilibrium transitions from relatively lower to relatively higher levels of participation. The inverse demand curve also has a kink when seller participation reaches the level that induces full participation by all bidders. Finally, the inverse demand curve is flatter than the inverse distribution of seller costs. This is because an increase in the fee not only reduces demand because fewer sellers have participation costs low enough to be willing to pay the higher fee; it also reduces demand because the decrease
in the number of sellers reduces the number of participating bidders, thus further reducing the value of the site to the sellers and the number of sellers who are willing to pay the fee — i.e., there are negative multiplier effects.

The above findings lead to several results from the perspective of the auction hosting site. First, unless participation on the bidder side of the market is inelastic (in our analysis this is the case in the so-called full bidder participation equilibrium), rent extraction by the auction hosting site from the sellers is severely limited due to the two-sided (negative multiplier) feedback effects of the entry equilibrium. Moreover, even if bidder participation is inelastic, the monopolist cannot extract surplus from the bidders through the use of a seller listing fee.

Second, when transitioning from a full bidder participation equilibrium to a partial participation entry equilibrium, the auction hosting site’s marginal revenue curve jumps down. Therefore, small changes in the environment (e.g., transaction costs, distribution of valuations, etc.) may have no change in the optimal fee whenever the optimal fee corresponds to the demand kink (though they will have an effect on seller participation rates and revenues).

Finally, small increases in the host site fee have dramatic effects if they lead to the elimination of the equilibrium with the hitherto highest participation and thus induce a jump to another entry equilibrium configuration. That is, the auction hosting site’s profit function not only has a kink, but it also has jumps (i.e., it is not continuous). Jumps are always downward and happen at the fee levels that correspond to the flat portions of the inverse demand, i.e., when a marginal change in the listing fee results in a jump to an equilibrium with discretely lower participation levels.

In conclusion, a monopoly auction hosting site has a smaller ability to extract surplus and, thus, the welfare consequences of monopoly are less severe. Moreover, traditional demand analysis can be very misleading both with regards to the computation of market surplus and also because the frequently imposed restriction of a (strictly) downward-sloping demand function is not appropriate. Finally, an auction host site is faced with two difficulties: first, due to multiplicity of stable equilibrium configurations in the entry equilibrium, it must ensure that coordination failures in the entry equilibrium are overcome; second, since a marginal increase in the listing fee may result in partial or complete collapse of the market as a jump to a lower entry equilibrium occurs, “trial-and-error” and “hill-climb” strategies as an approach to identify the optimal listing fee are unsuitable.

References