Can micro-credit bring development?☆

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Received 15 July 2005; received in revised form 12 July 2007; accepted 14 August 2007

Abstract

We examine the long-run effects of micro-credit on development in an occupational choice model similar to Banerjee and Newman (JPE, 1993). Micro-credit is modeled as a pure improvement in the credit market that opens up self-employment options to some agents who otherwise could only work for wages or subsist. Micro-credit can either raise or lower long-run GDP, since it can lower use of both subsistence and full-scale industrial technologies. It typically lowers long-run inequality and poverty, by making subsistence payoffs less widespread. Thus, an equity-efficiency tradeoff may be involved in the promotion of micro-credit. However, in a worst case scenario, micro-credit has purely negative long-run effects. The key to micro-credit’s long-run effects is found to be the “graduation rate”, defined as the rate at which the self-employed build up enough wealth to start full-scale firms. We distinguish between two avenues for graduation: “winner” graduation (of those who earn above-average returns in self-employment) and “saver” graduation (due to gradual accumulation of average returns in self-employment). Long-run development is not attainable via micro-credit if “winner” graduation is the sole avenue for graduation. In contrast, if the saving rate and self-employment returns of the average micro-borrower are jointly high enough, then micro-credit can bring an economy from stagnation to full development through “saver” graduation. Thus the lasting effects of micro-credit may partially depend on simultaneous facilitation of micro-saving. Eventual graduation of the average borrower, rather than indefinite retention, should be the goal of micro-banks if micro-credit is to be a stepping stone to broad-based development rather than at best an anti-poverty tool.

JEL classification: D31; D82; O11

Keywords: Micro-credit; Occupational choice; Long-run development; Poverty; Inequality

1. Introduction

Micro-credit has been called “one of the most significant innovations in development policy of the past twenty-five years.”¹ This movement aims to extend small amounts of capital to poor borrowers throughout the world, typically to facilitate income-generating self-employment activities. In the process it has popularized creative, perhaps

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☆ We are grateful for the encouragement and helpful comments of Dilip Mookherjee (the editor), two anonymous referees, Francesco Caselli, James Foster, Mercedes Garcia-Escribano, Maitreesh Ghatak, Xavier Gine, Hyeok Jeong, Andrew Newman, Kenichi Ueda, Clara Vega, Bruce Wydick, and seminar participants at Vanderbilt and NEUDC 2006. All errors are our own.

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doi:10.1016/j.jdeveco.2007.08.002

¹ Timothy Besley, quoted introducing Armendariz de Aghion and Morduch (2005). This book, Ghatak and Guinnane (1999), and Morduch (1999) provide broad introductions to the topic.
ingenious, lending techniques. For example, group lending employed by the Grameen Bank in Bangladesh and the Bank for Agriculture (BAAC) in Thailand, among others, has been shown theoretically to offer efficiency gains in contracting. Other aspects of micro-lending practice have also received attention, including its use of sequential loans and local information flows. Empirically, there is evidence that some of the mechanisms highlighted in theory are at work in practice. There is also some evidence for positive effects on household consumption and other outcomes from these programs.

Virtually all of the research thus far focuses on partial effects of micro-credit, often in a static context. The literature seems relatively quiet about the longer-run issues involved in micro-lending. On the empirical side, this may be explained by the relative youthfulness of the research agenda and the programs themselves. However, it does not seem too early to examine theoretically the potential long-run effects of micro-credit. Taking a step in this direction is the goal of the current paper.

Specifically, we study the long-run effects of micro-credit on development, measured by income per capita, inequality, and poverty. We use the occupational choice setting of Banerjee and Newman (1993). One advantage of this model for our purposes is its distinction between self-employment and entrepreneurship. Entrepreneurial activities are assumed to require a relatively large scale of capital and employment of wage laborers, while engaging in self-employment requires capital and only one’s own labor. The latter provides a fitting description of many activities funded by micro-credit.

We modify the model in two significant ways. First, we assume entrepreneurship is more efficient than self-employment. This creates a hierarchy of three technologies ranked by productivity and scale; in ascending order, they are subsistence, self-employment, and entrepreneurship. Second, we model micro-credit as the use of group lending to harness social pressure. This follows Besley and Coate (1995), and results in the borrower facing a higher cost of default. Assuming that micro-credit tends to target low-wealth borrowers, it increases access to self-employment but not entrepreneurship.

Thus micro-credit is modeled as a pure but limited improvement in the credit market. It opens up self-employment options to some agents who otherwise could only work for wages or subsist; but it does not directly affect access to full-scale entrepreneurship.

In the cases we analyze, micro-credit nearly always lowers poverty, defined as the number of people earning subsistence-level income. It does so by directly removing financial barriers and lifting some subsisters into self-employment. It can also do so more broadly by affecting the wage. Specifically, if enough households’ occupational choices are expanded to include self-employment, entrepreneurs must attract workers by paying a higher market wage, equal to the self-employment certainty-equivalent payoff. This allows all subsisters – even those without access to micro-credit – to escape from poverty.

Micro-credit also tends to lower inequality, by raising incomes of erstwhile poor subsisters and (potentially) lowering incomes at the top of the distribution by raising the wage paid by entrepreneurs.

Yet, though micro-credit is modeled as a pure credit market improvement, we find that it can raise or lower long-run output per capita. On the positive side, it lessens or eliminates use of the least-productive subsistence technology by removing financial barriers to self-employment. On the negative side, it can diminish use of the most productive entrepreneurial technology. Necessary for this to occur is that micro-credit raises the wage, which lowers entrepreneurial profits and can increase attrition of unsuccessful entrepreneurs from the entrepreneurial class — the set of households with sufficient wealth, given credit market imperfections, to run a full-scale firm. Since micro-credit can lower total income, it may therefore involve a long-run equity-efficiency tradeoff rather than being a “win–win” proposition.

Further, we find an exception even to the general rule that micro-credit lowers poverty in the long-run. In this perverse scenario, even though micro-credit initially raises income and reduces the number of subsisters, in the long run there are more subsisters, fewer entrepreneurs, lower aggregate income, and greater poverty. Key conditions for this scenario are that a) the rate of entrepreneurial attrition increases with the initial rise in the wage, and b) the odds of accumulating enough wealth from self-employment to start a full-scale firm are low relative to the odds of failing in both self-employment and entrepreneurship, where failing implies not having enough wealth to continue the venture (attrition). If these conditions hold, micro-credit can raise the wage only temporarily, since a critical mass of entrepreneurs (and hence labor demand) cannot be
maintained. Meanwhile, the depletion of the entrepreneurial class may be so significant that the economy ends up with fewer entrepreneurs and more subsisters in the long-run.

The main question considered in this paper is whether micro-credit can bring an underdeveloped economy to full development, defined (in this model) as the steady state with high wages and only the most efficient technology in use. The critical condition for full development is that there be a sufficiently large entrepreneurial class, that is, wealthy households who therefore have access to the most efficient technology and provide strong labor demand. Since micro-credit cannot directly enable full-scale entrepreneurship (by assumption), its long-run effects turn on the possibilities for “graduation” into the entrepreneurial class by households using the self-employment technology funded by micro-credit. Here “graduation” refers to sufficient accumulation of wealth by the self-employed (or those working for comparable wages) to enable entrepreneurship, operation of a full-scale firm.

The answer to our main question requires distinguishing between two potential avenues for graduation from self-employment to entrepreneurship. Households may eventually save enough wealth to graduate to entrepreneurship via sustained earning of normal (average) self-employment returns. This possibility we refer to as “saver graduation”. If saver graduation is possible, graduation is non-exceptional, though it may require saving over a significant period of time. On the other hand, if the savings rate and average returns in self-employment are jointly low, it may be impossible to accumulate enough wealth to graduate earning only normal self-employment returns. Graduation may still be possible, however, for those who earn supernormal (above-average) returns in self-employment — their high income can translate into sufficient wealth to start a full-scale firm. In this case, micro-credit can be viewed as opening self-employment opportunities to many individuals and allowing the market to select the particularly skilled or lucky ones for entrepreneurship. This type of graduation is referred to as “winner graduation”, since micro-credit essentially picks winners for graduation. In summary, winner graduation is open to those who earn relatively high returns in self-employment, while saver graduation comes from gradual accumulation of average self-employment returns.

If the self-employment technology funded by micro-credit allows winner graduation but not saver graduation, then micro-credit cannot bring development. That is, regardless of the frequency of supernormal returns in self-employment (holding the mean fixed), the entrepreneurial class cannot be sufficiently built. The reason is that, as the entrepreneurial class grows toward the key critical size, labor demand grows; consequently, a growing number of the agents who could be self-employed choose wage labor instead and earn a fixed wage equal to the average self-employment payoff. But wage labor does not involve the risk, upside or downside, that self-employment does. Thus, a declining segment of the middle class has the chance to earn supernormal returns on this road to development. As a result, growth in the entrepreneurial class stalls short of the critical mass. However, though a high winner graduation rate does not allow micro-credit to bring full development, it does result in higher long-run output. In fact, any mean-preserving spread that increases the frequency of both supernormal and subnormal returns in self-employment unambiguously raises long-run output.

On the other hand, if the self-employment technology funded by micro-credit allows for saver graduation, then sufficiently widespread micro-credit can bring development. Ultimately, it can move an economy from low output and subsistence wages to efficient technology and scale and to high wages, vanishing as an institution in the long-run. Necessary for this is that the rate at which households graduate via accumulation of normal self-employment returns be high enough relative to the rate of attrition out of the entrepreneurial class due to bad luck; if this condition holds, the entrepreneurial class grows until it reaches its critical value for development. In this scenario, micro-credit essentially breaks a subsistence-level poverty trap by offering access to more efficient capital intensity and technology; but breaking through the potential barriers to entrepreneurship must rely on jointly high savings rates and average self-employment incomes. Together, the results imply that if micro-credit raises incomes but leaves the average borrower trapped at a new, intermediate income level (non-poor but non-developed), then it cannot by itself be a stepping stone to full development; it is at best an anti-poverty tool.

Our paper has similarities with others in the literature. It borrows the basic framework from Banerjee and Newman (1993). Our focus on the effect of credit market improvements is different from theirs. Our results also differ due to a different technology assumption, which is discussed in the next section. We discuss briefly in Section 4.1 what effect the introduction of micro-credit (as modeled here) would have in their framework.

Matsuyama addresses similar issues in different models. Matsuyama (2007) shows that credit market improvements that increase access to non-frontier technologies may lower long-run efficiency, a phenomenon seen here. Matsuyama (2006) examines how the introduction of a moderately productive self-employment technology affects the set of potential steady states. He concludes...
that self-employment may raise or lower long-run income levels, similar to what we find here. These models differ from ours in significant ways. For example, they abstract from technological uncertainty, while uncertainty parameters figure prominently into the key conditions behind our analysis and results.

The policy conclusions we draw support a focus on “micro-finance” rather than on “micro-credit” exclusively. That is, the model identifies a novel complementarity between enabling saving and micro-lending: micro-lending can break a poverty trap, but it needs help from micro-saving to break any existing middle-income traps. Of course, the average level of technology made available by micro-lending is also critical; if low, graduation may be impossible even with high savings rates.\(^7\)

The model also suggests that micro-credit institutions ought to be oriented toward graduating their borrowers toward larger undertakings,\(^8\) and even to different credit institutions; the common focus on borrower retention may actually be a pitfall. A focus on graduation may make it hard to structure incentives for employees – to work themselves out of a job – and for borrowers – to repay loans when they are on the verge of graduating. Both of these incentive problems might be mitigated by information sharing among credit institutions, or by micro-credit institutions aspiring not to remain exclusively micro-credit institutions forever. These incentive problems seem worthy of greater exploration.

Section 2 introduces the basic static elements of the model, with Section 2.1 detailing the credit market and micro-credit. Section 3 introduces the dynamics. Section 4 presents the central examples and results: no graduation (Section 4.1), only winner graduation (Section 4.2), and only saver graduation (Section 4.3). Section 5 briefly discusses alternative approaches to this paper’s question, and Section 6 discusses policy implications and concludes.

2. Model

The model borrows significantly from Banerjee and Newman (1993, hereafter BN); major departures will be noted. At any date \(t\) there is a unit continuum of agents differing in wealth \(w\). Let \(G_t(w)\) be the fraction of agents with wealth less than \(w\) at time \(t\). Agents come to maturity at a random time distributed exponentially according to \(\lambda\), independent of wealth. At the instant of maturity they engage in all economic activity. Each mature agent is then replaced by, and leaves a bequest to, an offspring. The offspring is then dormant until his random instant of maturity is realized. This process gives rise to a stationary population, with identical wealth distributions for the active population and the entire population.\(^9\) We set \(\lambda \equiv 1\).

Agents have identical preferences over own consumption \(C\) and bequest \(B: B \equiv C^{1-\sigma}\). These preferences imply a fixed fraction of income \(s\) being used as a bequest. They also imply that indirect utility is linear in income, and thus agents are risk-neutral.\(^10\)

We assume a small open economy facing world (gross) interest rate \(\rho\). To restrict attention to finite wealth accumulation, we assume \(\sigma \rho < 1\).\(^11\)

All agents are endowed with one unit of labor and choose the occupation in which to spend it. There are three production technologies available to produce the only final good: (1) a subsistence technology which requires no capital and one unit of labor; (2) a self-employment technology which requires \(K_S\) units of the final good as an initial investment and one unit of labor; and (3) an entrepreneurial technology which requires a larger investment \(K_E (> K_S)\) and \(n \geq 1\) units of labor in addition to the entrepreneur’s.

These technologies result in four occupations from which to choose: subsistence, self-employment, entrepreneurship, and wage labor (for an entrepreneur). The subsistence technology produces \(v\) units of output. Thus an agent with endowment \(w\) who subsists ends life with a net worth of \(v + \rho w\). Similarly, choosing to be a wage worker results in earned income of \(v_n\), the market wage, and ending net worth of \(v_t + \rho w\).

The remaining two occupations involve uncertainty. Self-employment produces a random rate of return \(R_S\), where this and other rates of return are taken net of the opportunity cost of capital \(\rho\). For simplicity, \(R_S\) can take one of three values, high/supernormal \((R_{hS})\), medium/normal \((R_{mS})\), and low/subnormal \((R_{lS})\), with probabilities \(\pi_{iS}, i = h, m, l\), respectively. We assume that the expected

\(^7\) Relatedly, Kaboski and Townsend (2005) find that village-level institutions that promote savings and that offer training and advice register the greatest positive impacts. Karlan and Valdivia (2006) also find positive impacts from a training program attached to micro-lending.

\(^8\) This seems to support one of the well-known Grameen Bank ‘Sixteen Decisions’: “For higher income we shall collectively undertake bigger investments.” See Bornstein (1997, p. 97).

\(^9\) See BN (p. 278). As with them, our choice of overlapping generations in continuous time is to avoid jumps and overshooting.

\(^10\) See BN. Unlike them, we assume labor is supplied inelastically, which is inconsequential to the results.

\(^11\) If \(\sigma \rho > 1\), any positive amount of wealth grows without bound. This eventually makes the credit market restrictions discussed below irrelevant (with fixed technology levels), and any economy will converge to full development regardless of initial wealth levels and distribution. In a version of the model with exogenously growing technology and capital requirements, the economy’s development path would depend on relative paces of growth of technology, capital requirements, and wealth, if \(\sigma \rho > 1\).
Entrepreneurship generates a random (net) rate of return $\bar{R}^E$. $\bar{R}^S$ can take two values, high ($R^E$) and low ($\tilde{R}^E$), with probabilities $\pi^E_i$, $i = h, l$, respectively. Let $\tilde{R}^E$ denote the expected return. An entrepreneur with wealth $w$ ends with net worth of $\tilde{R}^E K^E - n v_i + \rho w$, after paying the wages of $n$ workers.

Let $\tilde{\nu} = \tilde{R}^S K^S$ and $\bar{\nu} = R^E K^E / (n + 1)$ be the expected output per unit of labor of the self-employment and entrepreneurial technologies, respectively. We assume that

$$\nu < \tilde{\nu} < \bar{\nu}. \quad (A1)$$

This assumption creates a hierarchy of three technologies, with entrepreneurial production most efficient, subsistence least efficient, and self-employment in the middle. Assumption (A1) is critical to our results and is our key departure from BN. (However, it is quite similar to the technology structure in Matsuyama, 2006, where self-employment is assumed less productive than entrepreneurship.) There are essentially two production technologies in BN;12 three technologies are necessary here given our construction of micro-credit as a credit market improvement that increases access to better, but not best, technology and capital intensity (see next section).

This three-technology hierarchy can be justified on several grounds. One rationale for this hierarchy is that capital intensity is a key to productivity. It has been assumed that $K^S < K^E$; if further $K^S < K^E / (n + 1), \text{13}$ then per-person capital requirements across occupations vary monotonically with efficiency. A second, complementary rationale for entrepreneurship’s greater efficiency is that production organized within a firm is often more efficient than production organized across autonomous individuals. This can be due to scale economies and/or greater scope for specialization within firms.

Assumption (A1) seems to accord broadly with several empirical facts. First, the large share of production accounted for by wage labor and the capital of relatively large firms in developed countries makes it empirically plausible that entrepreneurship is in general more productive than self-employment (defining both occupations as we do here). Scale economies and/or easier within-firm specialization seem to constitute an empirical regularity in the many sectors in which large firms dominate. Second, while micro-credit improves micro-borrowers’ access to capital and technology, it does not seem in general to allow them to reach optimal capital scale and technology levels. Most micro-loans are fractions of domestic GDP/capita, and micro-lenders often appear to ration funds, perhaps aiming for broader outreach or to provide repayment incentives via promises of larger future loans. It seems clear that most micro-credit funded enterprises work with less capital and less advanced technology than the average developed country worker. In this context, micro-credit may be useful to a stagnant economy as a stepping stone to fully efficient production, but it does not represent the ideal long-run solution. The three-technology hierarchy will allow us to define micro-credit in a way that matches this stylized fact; micro-credit will open access to self-employment, thereby increasing capital intensity and technology levels, but not to fully developed levels.

The first-best of this economy is achieved when exclusively entrepreneurial technology is used, that is, when there are $1 / (n + 1)$ entrepreneurs and $n / (n + 1)$ workers. A perfect credit market would give this outcome, but we assume there are imperfections.

2.1. Credit market and micro-credit

The credit market is competitive, leading to a (gross) interest rate of $r = \rho$ charged on all loans that involve no default risk. Following BN, we assume a borrower always can repay but may choose not to after production. Specifically, an agent who borrows $K$ can refuse to repay at the cost of facing a (non-monetary) punishment $F$ with probability $q$ and losing whatever collateral was pledged, $C$. Borrowers then face the choice of whether to default, saving principal and interest $rK$ but losing collateral $rC$ and facing expected penalty $qF$. If $rK > rC + qF$, i.e. $C < K - qF / r$, an agent will choose to default.

In equilibrium, lenders will demand sufficient collateral so that no default occurs. Thus $r = \rho$, and collateral $C \geq w^* \equiv K^S - qF / \rho$ is required from any agent seeking to become self-employed and $C \geq w^{**} \equiv K^E - qF / \rho$ from any agent seeking to become an

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12 Entrepreneurship is not a separate production technology in BN, but a separate institution: one agent monitoring $n$ other agents who each use the self-employment technology. In fact, in BN entrepreneurship is less productive than self-employment, since it uses up the entrepreneur’s unit of labor in non-productive monitoring; however, they do not stress the efficiency differences between these occupations. It seems to us that their technology assumptions are not meant to take a stand on the relative efficiencies of self-employment and entrepreneurship, but are made as streamlined as possible while still allowing their points about institutions and history dependence to be made.

13 This can be assumed without affecting any results, but is not needed for any of them given Assumption (A1).
entrepreneur.\textsuperscript{14} We assume the credit market imperfection is severe enough \((qF^\prime)\text{ small enough}\) so that \(w^* > 0\).

The credit market imperfection divides the population into three classes that differ in occupational possibilities. The wealthiest class, those with \(w \geq w^{**}\), can choose any occupation. We call them the “entrepreneurial class”, or for convenience, the “upper class” and let \(P_t^E \equiv l - G(w^{**})\) denote their measure at date \(t\). Similarly, the “lower class” with wealth \(w < w^*\) can only subsist or work for wages; its measure is \(P_t^L \equiv G_t(w^*)\). Finally, the residual “middle class” can choose any occupation but entrepreneurship; its measure is \(P_t^M \equiv G_t(w^{**}) - G_t(w^*)\).

In this context, micro-credit will be defined as a pure credit market improvement that improves low-wealth borrowers’ access to credit. To do so, we follow the literature that views micro-credit as innovative use of joint liability contracts to enable more efficient lending; specifically, as in Besley and Coate (1995, hereafter BC), micro-credit makes default less attractive by harnessing social penalties.

Assume the lender pairs borrowers together in groups of two and makes each one responsible for the other’s repayment. Specifically, if borrowers \(i\) and \(j\) are paired together and \(j\) defaults, \(i\) faces penalty \(F^\prime\) with probability \(q^F\); and vice versa. This penalty is additional to any potential penalty incurred for his own default \((F)\).

The key assumption, adapted from BC, is that an informal, social penalty is imposed on a borrower who defaults, since default led to his partner facing additional penalty \(q^F \cdot F^\prime\). The social penalty could involve ostracism, withholding of informal aid or insurance, and/or the breaking of productive relationships. Let \(\phi\) be the magnitude of the social penalty imposed on a borrower who defaults.

Repayment calculus changes with micro-credit. If a borrower chooses to default rather than repay, he saves principal and interest \(pK\), loses collateral \(pC\) and faces expected penalty \(qF\); as before; in addition, there is the social penalty for defaulting, \(\phi\). (He may also be penalized \(q^F \cdot F^\prime\) by the lender for his partner’s default, but this is independent of his own repayment decision.) Thus if \(pK > pC + qF + \phi\), i.e. \(C < K - (qF + \phi)/\rho\), default will occur. To acquire a loan \(K\), collateral at least \(K - (qF + \phi)/\rho\) is required; this collateral requirement is directly lowered by the social penalty, \(\phi\), which augments the official penalty \(qF\). Group lending therefore represents a pure credit market improvement here.

Micro-credit with group lending and social penalties could in principle lower both wealth cutoffs, \(w^*\) and \(w^{**}\) (defined above), in the amount \(\phi/\rho\). Empirically, however, micro-credit tends to be targeted to low-wealth borrowers rather than borrowers of all wealth levels. Further, the types of projects undertaken by micro-borrowers are commonly small-scale micro-enterprises rather than full-scale firms. In order to match the empirical reality of micro-credit, we assume that only the wealth cutoff for self-employment, \(w^*\), is lowered, and that access to entrepreneurship, \(w^{**}\), is not affected. This assumption is critical to results, but it will be straightforward to see how results change without it.

There are several potential micro-foundations for this assumption. First, it is plausible that dependence on informal insurance networks, which can be the basis for informal penalties (as in Paal and Wiseman, 2005), declines with wealth. Group lending would then offer little or no enforcement enhancement among richer borrowers, who are less vulnerable to informal penalties. Second, one could assume the social penalty is occupation-specific, positive for self-employment and zero for entrepreneurship. This would be justified if the entrepreneurial technology had to be employed in an anonymous or high-mobility urban context, while the self-employment technology could be operated in a village, with tight social networks. Banerjee and Newman (1998) tell a related story. Third, other factors could cause the attractiveness of group loans to decline with wealth. Madajewicz (2005) shows in a hidden action environment how joint liability loans may not be optimal for higher-wealth borrowers since the increased risk from liability for partners’ loans dominates the benefit of increased monitoring incentives. Incorporating one or more of these mechanisms is beyond the scope of this paper, and we directly assume that the social penalties harnessed by group lending affect \(w^*\) but not \(w^{**}\).

In summary, micro-credit is modeled as a pure but limited improvement in the credit market. It has the immediate effect of growing the middle class \(P_t^M\) at the expense of the lower-class \(P_t^L\), without changing the upper-class \(P_t^E\).

2.2. Labor market equilibrium

Occupational choices depend on the equilibrium wage. The wage in turn depends on the wealth distribution as summarized through \(P_t^U, P_t^M, \) and \(P_t^E\),

\textsuperscript{14} In this model, an agent partially or fully self-financing is equivalent to an agent financing the full amount externally, since wealth is equally effective as collateral or productive capital. We can thus think of all agents as financing externally.
since these determine labor supply and demand; these are graphed in Fig. 1.

Labor demand can only arise from the upper class, who are able to become entrepreneurs. Given Assumption (A1), it is straightforward to show that entrepreneurship gives the best payoff as long as \( v < \bar{v} \). This critical wage \( \bar{v} \) equals the wage labor payoff, \( v_p \), with the expected entrepreneurial payoff, \( R^E K^E - n v_p \). If \( v > \bar{v} \), wage labor gives the highest payoff. Thus, since each entrepreneur hires \( n \) workers, total demand will be \( n P_t^L \) if \( v < \bar{v} \); 0 if \( v > \bar{v} \); and any amount in \( [0, n P_t^L] \) if \( v = \bar{v} \), since the upper class are then indifferent between wage labor and entrepreneurship.

Turning to labor supply, note that there are three levels for the wage that put wage labor on par with the three respective occupations: \( v \) for subsistence, \( \bar{v} \) for entrepreneurship, and \( \hat{v} \) for self-employment. Given the occupational restrictions placed by the credit market, labor supply is then zero when \( v < \bar{v} \); \( P_t^L \) when \( \bar{v} < v < \hat{v} \); \( P_t^L + P_t^M \) when \( v < \hat{v} < v_t \); and 1 when \( \hat{v} < v_t \). If \( v_t \) equals \( \bar{v}, \hat{v} \), or \( v \), a mass of agents is indifferent between labor and some other occupation, and labor supply can take any value between its limits from below and above.

Combining supply and demand leads to one of the following three equilibrium wages:\footnote{In contrast, there are two potential equilibrium wages in BN, due to their assumption that self-employment is more productive than entrepreneurship.}

- If \( n P_t^L < P_t^L \), the equilibrium wage rate is \( v_t = v \).
- If \( P_t^L < n P_t^L < P_t^L + P_t^M \), the equilibrium wage rate is \( v_t = \bar{v} \).
- If \( P_t^L + P_t^M < n P_t^L \), that is, \( P_t^L > 1/(1+n) \), the equilibrium wage rate is \( v_t = \hat{v} \).

In the first case, the pool of potential entrepreneurs is too small relative to the population whose only options are wage labor or subsistence. The wage is bid down to subsistence level. All upper-class agents take advantage of the low wages to become entrepreneurs; all middle-class agents choose self-employment; and some of the lower-class work in firms \( n P_t^L \) while the rest subsist \( (P_t^L - n P_t^L) \).

In the second case, there are enough potential entrepreneurs to need some middle-class workers, so they pay \( \hat{v} \). All upper-class agents become entrepreneurs, all lower-class and some middle-class agents become workers, and the rest of the middle-class \( (1 - (n+1) P_t^L) \) become self-employed.

In the third case, the potential entrepreneurial pool is so large that it needs to attract some of its own members to work. The equilibrium wage makes working in a firm as attractive as running one. All lower- and middle-class agents, and some upper-class agents, become workers and receive the same labor income\footnote{“Labor income” and “income” are used synonymously with the occupation-specific component of income. They exclude interest income \( \rho w \).} as entrepreneurs (in expected value). Since everyone in this economy is working in a firm with the entrepreneurial technology, the highest aggregate output is achieved; and the high wages guarantee perfect equality in (expected) income.

The wealth distribution thus determines the current wage rate, which determines occupations, incomes, and the future wealth distribution.\footnote{The cases where \( n P_t^L \) exactly equals either \( P_t^L \) or \( P_t^L + P_t^M \) can see any of a range of wages. We assume for simplicity that the highest possible wage prevails in these cases.} These dynamics are analyzed next.

3. Dynamics

In Section 3.1, we present the family wealth dynamics under each of the three potential equilibrium wages discussed above. Section 3.2 considers aggregate wealth dynamics.

3.1. Family dynamics

An agent passes a fraction \( s \) of his ending net worth to his offspring. Let \( V(v_t, w) \) be the occupation-specific income earned by an agent of wealth \( w \) who comes to maturity when the wage is \( v_t \); these values are readily calculated from the discussion of the three cases in Section 2.2. The bequest his offspring receives, i.e. his offspring’s wealth, is then \( w' = s [V(v_t, w) + \rho w] \).

Recall from Section 2.2 that \( P_t^L \) and \( P_t^M \) are the only features of the wealth distribution needed to calculate the wage \( v_t \) at any time \( t \). However, generally the entire
wealth distribution is needed in order to track the evolution of $P_t^U$ and $P_t^L$ over time. For tractability, we follow BN and restrict analysis to cases exhibiting the following Markov property: the wealth class and occupational choice of any agent, together with the market wage, fully determine probabilities of transition to future wealth classes. This “BN Markov assumption” dramatically reduces the state-space, to the two-dimensional simplex ($P_t^U$, $P_t^L$).

One set of parameter values that allows for this dimensional reduction is displayed in Fig. 2. There are three panels, one for each possible market wage. The first panel corresponds to $v_t = v$. At this wage the lower-class work or subsist to earn $v$. The graph depicts them as stuck in the lower class: given the saving and interest rates, they can never accumulate enough to finance self-employment.\footnote{The condition for this is $s(v + \rho w^*) < w^*$, that is, $sv/(1 - s\rho) < w^*$. This is satisfied when the savings rate, return to savings, and subsistence wages are jointly low enough compared with the wealth cutoff for operating a micro-enterprise, which reflects the degree of imperfection in the credit market.}

The middle class are all self-employed. The graph depicts them remaining in the middle class if they earn normal returns, graduating to the upper class if they receive supernormal returns (i.e. “win”), and falling into the lower class if they receive subnormal returns. Finally, the upper class work as entrepreneurs. The graph depicts them always earning enough to stay in the upper class: even if unlucky, the wage bill is low enough to keep them wealthy.

The second panel applies to $v = \bar{v}$. There, wages are so high that both low- and middle-class families transit to the upper class. The down side is that unlucky entrepreneurs now transit to the lower class after paying the high-wage bill. Results would not change if unlucky entrepreneurs transited to the middle class — everyone earns the same high (expected) income regardless, and the dynamics will guarantee that the wage remains high if it ever gets high (since $\pi_t E > n$).

In summary, the main restriction in Fig. 2 is the BN Markov assumption discussed above. Beyond this, a few additional restrictions are imposed. Regarding the lowest class, the main restriction is that the subsistence wage is too low to provide poor households any upward mobility. This low-wage induced poverty trap seems empirically plausible, and such traps are not uncommon in models like this (including BN). Regarding the middle class, the main assumption is that only “winners” can graduate from the middle class, while the mean self-employment outcome leaves a household in the middle class. This creates a second, middle-income trap for the average self-employed household, which is the one assumption critical to Proposition 1 below. This middle-income trap is removed in the discussion of “saver graduation” in Section 4.3 (Fig. 6). The middle class are otherwise modeled flexibly: mobility from self-employment to any
class is allowed, and we will address the entire parameter space governing these transition probabilities. Finally, assumptions on the upper class guarantee that successful entrepreneurs remain in the upper class regardless of the wage — this is necessary for the possibility of a fully developed steady state. The most substantive assumptions address what happens to unsuccessful entrepreneurs: namely, they remain wealthy if wages are low, drop to the middle class if wages are moderate, and drop to the lowest class when wages are high. As noted, these assumptions do not drive either of the propositions below. However, while the specifics are not important, the idea that higher wages lead to greater downward mobility of unsuccessful entrepreneurs is critical to the existence of an equity-efficiency tradeoff involved in micro-credit.

3.2. Aggregate dynamics

Given the family wealth evolution described in the previous section and Fig. 2, we can accomplish our ultimate objective: tracing the economy-wide wealth and income distributions. Each of the panels in Fig. 2 leads to a pair of first-order differential equations in $P_t^L$ and $P_t^U$. For example, when $v_i=\bar{\nu},$

$$
\begin{align*}
\dot{P}_t^U &= \pi_h^U (1 - P_t^L - P_t^U) \\
\dot{P}_t^L &= \pi_l^L (1 - P_t^L - P_t^U).
\end{align*}
$$

These equations reflect the fact that when $v_i=\bar{\nu}$ the only class mobility comes from the middle class, all $(1 - P_t^L - P_t^U)$ of whom are self-employed. A fraction $\pi_h^U$ of the middle class who are active at date $t$ are winners and graduate to the upper class, causing $P_t^U$ to grow. Similarly, a fraction $\pi_l^L$ are unlucky and drop to the lower class, causing $P_t^L$ to grow.

When $v_i=\bar{v},$

$$
\begin{align*}
\dot{P}_t^U &= \pi_h^U (1 - (n+1)P_t^U) - \pi_l^U P_t^U \\
\dot{P}_t^L &= \pi_l^L (1 - (n+1)P_t^U) - P_t^L.
\end{align*}
$$

Again, the middle class provides mobility, but only those who are self-employed. Recall that the middle class divides itself between wage labor and self-employment when $v_i=\bar{\nu}.$ In fact, the entire population is running a firm or working in one, except the self-employed middle class. Since there are $P_t^U$ firms, the measure of self-employed must be $1-(n+1)P_t^U$. Mobility also comes from active lower-class agents; at this wage they immediately move up to the middle class (hence the $-P_t^L$ term in $\dot{P}_t^U$). Active upper-class agents whose firms perform poorly now drop out of the upper class (hence the $-\pi_l^L P_t^U$ term in $\dot{P}_t^U$).

When $v_i=\bar{\nu},$

$$
\begin{align*}
\dot{P}_t^U &= (1 - P_t^U) - \frac{\pi_l^U}{n+1} \\
\dot{P}_t^L &= -P_t^L + \frac{\pi_l^L}{n+1}.
\end{align*}
$$

The first term in $\dot{P}_t^U$ is because everyone not in the upper class $(1-P_t^U)$ transits there, due to the high wages. The second term reflects the fraction of unlucky
entrepreneurs who drop to the lower class. The maximum number of entrepreneurs in the economy is $1/(n+1)$; this is also the actual number whenever the wage is this high. The first term in $P_t^L$ is because lower-class agents transit upward; the second term captures the unlucky entrepreneurs.

The economy can be described at any time $t$ by a point on the two-dimensional simplex $(P_t^L, P_t^U)$; see Fig. 3. This triangle is subdivided into three “regions” corresponding to the three equilibrium wages (see Section 2.2). Points in the upper triangle where $P_t^L > 1/(n+1)$ involve the high-wage $v_t = \bar{v}$. Points in the lower right triangle where $P_t^L < P_t^L/n$ involve the low-wage $v_t = \bar{v}$. The remaining triangle involves the medium wage, $v_t = \bar{v}$. The economy’s initial location thus determines which of the dynamics (Eqs. (1)–(3)) apply at the outset and whether the economy may ultimately progress to higher wages and greater efficiency.

4. Micro-credit and long-run development

We illustrate the possibilities opened up by micro-credit through several examples. Our strategy is to compare the long-run outcomes of two initially identical economies, only one of which experiences the introduction of micro-credit.

4.1. Dynamics with no graduation

Consider the simple case in which uncertainty in self-employment returns is eliminated: $\pi_s^X = \pi_t^X = 0$. This implies that the self-employed middle class cannot graduate upward.

Fig. 3 shows the evolution of the wealth distribution. We see from Eq. (1) that $P_t^U = P_t^L = 0$ in the $v$-region. Each class is absorbing, so that a wealth distribution in this region stagnates at the same location forever; see point $SS$ in Fig. 3. A wealth distribution in the $v$-region, however, converges to the origin (point $SS$), since both $P_t^U$ and $P_t^L$ are negative (Eq. (2)).

The $v$-region thus involves shrinking upper and lower classes (both due to higher wages) and an absorbing middle class; in the end, all are self-employed. In the $v$-region, any distribution converges to point $SS$ defined as $(P_t^L, P_t^U) = (\frac{1}{n+1}, 1 - \frac{1}{n+1})$. All agents are employing the most efficient technology and have the same expected income. Here, each region of the simplex is self-contained, that is, there are no paths from one $v$-region to another.

We can compare the (labor) income distributions of these possible long-run economies in terms of both average level, inequality, and poverty. To facilitate discussion about poverty, we assume the poverty line $p$ satisfies

$$v < p < \hat{v}.$$  \hspace{1cm} (A2)

As mentioned, economy $\bar{SS}$ involves use of only the most efficient technology; further, everyone earns the high-wage $\bar{v}$ (in expected value), so there is no poverty or inequality. This ideal outcome is referred to as “development”. Economy $SS$ also has perfect equality and no poverty since everyone is self-employed. Average income is of course lower, equal to $v$. Finally, economy $SS$ clearly has higher poverty and inequality than the others, as the lower class earn $v$, the middle class earn $\hat{v}$, and the upper class earn $\hat{v}$. Average income is a weighted average of the three technologies’ productivities. It is thus clearly lower than at $SS$, but can be higher than at $SS$, since it involves both more entrepreneurship and more subsistence. In particular, there is a critical line from the origin, with slope less than $1/n$ and therefore lying in the $v$-region. If $P_t^U / P_t^L$ is high enough so that $SS$ lies above this line, then $SS$ has higher income than $\bar{SS}$; and vice versa.\(^{20}\)

Now imagine two economies at $SS$ and the introduction of micro-credit in one. Can this financial development bring the economy to development? Recall from Section 2.1 that the introduction of micro-credit is equivalent to a decline in $P_t^L$ mirrored by a rise in $P_t^U$, with $P_t^U$ unchanged. In Fig. 3 this would be represented by a leftward shift from point $SS$. There are two qualitative cases to consider. First, micro-credit may impact few enough people that the treated economy remains in the $v$-region (pictured as the shift to $SS'$). The only change in this economy is that the agents impacted by micro-credit move from subsistence or wage labor to self-employment. Clearly, micro-credit leads to higher average income and lower poverty. The treated and untreated economies’ Lorenz curves cross, though, so inequality is less clear; but the treated economy’s generalized Lorenz curve dominates the untreated economy’s.\(^{21}\) Second, micro-credit may impact enough people, that is, improve enough agents’ productive options, to raise the wage to $\hat{v}$ (pictured as the shift to $SS''$). As analyzed above, the treated economy will converge to steady state $SS$, getting

\(^{20}\)Income at $SS$ can be written as $(n+1)P_0^U \times \bar{v} + P_0^L \times \bar{v} + (P_0^U - nP_0^L) \times v$. The critical line in the $v$-region, along which average income is $v$, is then $p = p^*/(1 + v/n)$.

\(^{21}\)The generalized Lorenz curve is the Lorenz curve multiplied by the mean income. Generalized Lorenz dominance is the extension of Lorenz dominance to generalized Lorenz curves.
rid of inequality and poverty, but potentially lowering total output if the initial point $SS$ was high enough.22

In summary, the introduction of micro-credit lowers poverty and inequality (though if it fails to raise the wage, there are exceptions under Lorenz dominance but not generalized Lorenz dominance). Micro-credit may or may not raise total income, however, since it can reduce the use of the most productive, entrepreneurial technology; if sufficiently widespread, it raises the wage and lowers entrepreneurial profits, causing attrition from the entrepreneurial class. Thus micro-credit may involve an equity-efficiency tradeoff. In any case, micro-credit cannot bring full development when $\pi_h = \pi_l = 0$.23

4.2. Dynamics with winner graduation

The development prospects of micro-credit may be enhanced if it provides a substantial possibility of graduating from self-employment to entrepreneurship. Assume now that with probability $\pi_h > 0$ an agent experiences supernormal returns allowing his offspring to graduate to entrepreneurship, while with probability $\pi_l > 0$ an agent experiences subnormal returns and puts his offspring in the lower class.

Fig. 4 shows the evolution of the wealth distribution. The $v$-region sees the same dynamics (convergence to $SS$) as in Section 4.1, since $\pi_h$ and $\pi_l$ do not figure into Eq. (3). Dynamics in the $v$-region now reflect a shrinking middle class: there remains no mobility out of the upper and lower classes, but a constant flow into each from the middle class. The economy moves upward and to the right along a linear trajectory whose steepness depends on the relative rates of graduating upward out of self-employment ($\pi_h$) and dropping into poverty out of self-employment ($\pi_l$). Specifically, Eq. (1) gives its slope $\dot{P}_U / \dot{P}_L$ as

$$a = \frac{\pi_h}{\pi_l}.$$
These starting points have relatively high $P_{0}^{U}$ compared to $P_{0}^{L}$; they correspond to the non-shaded area of the $v$-region in Fig. 4. The remaining initial distributions, in the shaded area, will converge to points on the upper right leg of the $v$-region where $P_{t}^{L} + P_{t}^{U} = 1$; one example is denoted by $SS$.

It can be verified that all distributions in the $\hat{v}$-region will converge to

$$
\left( P_{SS}^{L}, P_{SS}^{U} \right) = \left( \frac{\pi_{l}^{E}}{a} \frac{1}{n + 1 + \frac{\pi_{l}^{E}}{\pi_{b}^{E}}} \frac{1}{n + 1 + \frac{\pi_{l}^{E}}{\pi_{b}^{E}}} \right). \tag{4}
$$

Fig. 4 depicts the phase diagram and steady state denoted by $SS$.

Can micro-credit bring an underdeveloped (low- or mid-wage) economy to full development? No. Fig. 4 makes clear that a leftward shift of any economy may eventually bring the economy to the mid-wage steady state $SS$, but it cannot bring it to the high-wage steady state $SS$. This is also clear from Eq. (4): $P_{SS}^{L}$ is less than the critical value of $1/(n + 1)$ no matter what the value of $\pi_{b}^{E}$.

These arguments rely on the specific class transitions of Fig. 2. However, this result holds for any class transitions that satisfy the BN Markov assumption, Assumption (A1), and the assumption that only winners graduate, that is, that average returns in self-employment ($\hat{v}$) are not sufficient to allow graduation to the upper class:

$$
\hat{s}(\hat{v} + \rho w^{**}) < w^{**}; \quad \text{i.e.} \quad \frac{s \hat{v}}{1 - s \rho} < w^{**}. \tag{A3}
$$

**Proposition 1.** Under the BN Markov assumption and Assumptions (A1) and (A3), micro-credit cannot bring an underdeveloped (low- or mid-wage) economy to full development.

**Proof.** See Appendix.

This result is somewhat surprising; one might think that a sufficient proportion of self-employed winners graduating could more than compensate for attrition of entrepreneurs and raise the number of entrepreneurs $P_{t}^{U}$ to its critical value $1/(n + 1)$. But the intuition is as follows. Micro-credit can raise the wage to $\hat{v}$ but not to $v$. In the $\hat{v}$-region, if the number of entrepreneurs $P_{t}^{U}$ were to increase toward the critical value $1/(n + 1)$, labor demand would grow; as a result, more of the middle class would engage in wage labor rather than self-employment (two occupations between which they are indifferent). In the limit, if $P_{t}^{L}$ were to approach $1/(n + 1)$, the number of self-employed would approach zero as all the middle class worked for wages instead. But, upward mobility into entrepreneurship comes only from the self-employed; workers earn the mean self-employment payoff with certainty, which by assumption does not allow graduation. Thus, mobility into entrepreneurship vanishes as the number of entrepreneurs approaches the critical value.

This result is robust to any assumptions about uncertainty in the three occupations (including $\pi_{l}^{E}=0$ and/or $\pi_{l}^{E}$, near 1). The critical assumption is the mid-wage trap, i.e. the inability of households earning average returns in self-employment ever to graduate to the entrepreneurial class. The model implies, then, that if micro-credit promotes technologies that cannot raise the wealth level of the average borrowing household enough to fund larger endeavors – via accumulating saved earnings, over a period of time that admittedly may be a generation or more – then micro-credit is at best an anti-poverty tool rather than a stepping stone to broader development.

While not a panacea in this case, micro-credit can have positive long-run effects. Economies starting in the shaded area of the $v$-region in Fig. 4 will have their long-run outcomes altered. (All other economies have the same long-run destiny with or without micro-credit.) As in Section 4.1, there are two qualitative cases to consider. First, micro-credit may impact few enough people that the treated economy remains in the shaded area of the $v$-region (pictured as the shift from $I$ to $I'$). Some of the agents lifted to self-employment eventually graduate to become entrepreneurs, leaving the long-run treated economy with more entrepreneurs and fewer subsisters ($SS'$ instead of $SS$). Clearly, average income is higher and poverty is lower due to micro-credit. The Lorenz curves of the treated and untreated economies cross, but the treated economy’s generalized Lorenz curve dominates the untreated economy’s.

Second, micro-credit may impact enough people to move the treated economy out of the shaded area (the shift from $I$ to $I''$) and thus ultimately to $SS$. Micro-credit would then clearly have lowered poverty, since everyone earns at least $v$ in the treated economy ($SS$) while some agents earn $v$ (as workers or subsisters) in the untreated economy ($SS$).

In this case, the effect on long-run average income can be positive or negative. Average income in the untreated economy, $y_{SS}$, is a convex combination of the entrepreneurial and subsistence productivities, while average income in the treated economy, $y_{SS}$, is a convex
combination of the entrepreneurial and self-employment productivities:

\[
y_{\text{SS}} = \hat{v} \cdot P_{\text{SS}}^U(n + 1) + v \cdot \left[1 - P_{\text{SS}}^U(n + 1)\right]
\]

\[
y_{\text{SS}} = \hat{v} \cdot P_{\text{SS}}^U(n + 1) + v \cdot \left[1 - P_{\text{SS}}^U(n + 1)\right].
\]

Both equations reflect the fact that the total population working in firms is \(P_{n+1}^U\). While \(y_{\text{SS}}\) is fixed strictly below \(\hat{v}\) since \(P_{\text{SS}}^U\) is strictly less than \(1/(n+1)\) (see Eq. (4)), \(y_{\text{SS}}\) can be arbitrarily close to \(\hat{v}\), since \(P_{\text{SS}}^U\) can be arbitrarily close to \(1/(n+1)\), depending on initial conditions. Thus there is a range of steady states \(\text{SS}\) in the \(v\)-region that involve higher average income than \(\overline{\text{SS}}\). That is, micro-credit can lower average income even as it lowers poverty.

The intuition is the same as in Section 4.1. On the positive side, micro-credit reduces use of subsistence technology in favor of self-employment technology; on the negative side, it raises the wage and the amount of attrition out of entrepreneurship, potentially lowering use of the entrepreneurial technology in the long-run.

Both steady states involve inequality, and inequality comparisons can be ambiguous. What is clear is that when \(\overline{\text{SS}}\) has higher income than \(\text{SS}\), \(\overline{\text{SS}}\) also generalized Lorenz dominates \(\text{SS}\) (though it may be Lorenz dominated). When \(\overline{\text{SS}}\) has lower income than \(\text{SS}\), the generalized Lorenz curves cross; but \(\overline{\text{SS}}\) Lorenz dominates \(\text{SS}\) if

\[
\frac{\hat{v}}{v} \left(1 + \frac{\hat{v} - v}{\hat{v} - v}\right) \geq \frac{n + 1}{n}.
\]

Thus, \(\overline{\text{SS}}\) achieves higher income without sacrificing equality, or it achieves lower income but a more equal distribution (under condition (A4)).

We finally note that risk-taking in self-employment gives long-run benefits to economies transiting toward \(\text{SS}\). Consider a mean-preserving spread in returns that raises \(\pi_{\text{SS}}^S\) and \(\pi_{\text{SS}}^G\) while holding fixed their ratio \(a\) as well as \(R_{\text{SS}}^S\), \(R_{\text{SS}}^G\), and \(R_{\text{SS}}^L\). This raises \(P_{\text{SS}}^U\) (see Eq. (4)), which unambiguously raises output (see Eq. (5)). The resulting income distribution generalized Lorenz dominates the original. This result suggests that encouraging risk-taking in self-employment activities rather than failsafe approaches (which deliver high repayment rates) may improve long-run outcomes.

Micro-credit cannot bring full development if it relies only on winner graduation for mobility into the entrepreneurial class; but it can lower poverty and inequality and, under some conditions, raise total income. We next show that micro-credit can have a purely negative long-run impact when \(a\) is small.

---

\[26\] Even without condition A4, \(\overline{\text{SS}}\) is never Lorenz dominated by \(\text{SS}\) when \(y_{\text{SS}} > y_{\text{SS}}\); but there may be low-wage steady states (with high enough \(P_{\text{SS}}^U\)) whose Lorenz curves cross that of \(\overline{\text{SS}}\).

\[27\] This holds when graduation is limited to winner graduation; the presence of saver graduation (analyzed in Section 4.3) could eliminate the economy-wide gains from risk-taking. Also, including risk aversion in the model might alter this conclusion; the net effect on long-run welfare from the increase in both risk-taking and income might be ambiguous. If the lender absorbed a sufficient amount of project risk, however, the welfare effect would be positive. We discuss risk aversion in more detail in Section 5.
4.2.2. Dynamics with a small
Assume that winner graduation is relatively rare, specifically \( a < \pi_l / n \).\(^{28}\) In this case, there is no steady state in the \( v \)-region; all economies there eventually drop into the \( v^* \)-region, as is evident from Fig. 5. In much of the \( v \)-region, the middle class is growing at the expense of both the lower and upper classes. But, under the assumed parameter values, the upper class continues to decline even as the lower class begins to rebound. This lower-class resurgence occurs after the middle class has gotten so big that downward mobility out of it dominates upward mobility into it from the lower class. The continuing decline of the upper class even when the middle class is big is because the rate of graduation from middle to upper class, \( \pi_h \), is so low relative to the rate of falling out of the upper class, \( \pi_l \). Eventually, the upper class decline and lower-class resurgence is sufficient to lower the wage to \( v \). At this point, \( v \)-region dynamics take over and the steady state is on the upper right leg of the \( v \)-region.

Under these parameters, micro-credit can make an economy unambiguously worse off in the long-run. It can do so, paradoxically, only if it is widespread enough to effect a transition to the mid-wage \( v \). It provides the economy a temporary spell of higher wages (\( v = \tilde{v} \)), depleting both upper and lower classes; but if the decline in the upper class is sufficiently dominant, the mix of upper and lower-class agents when the wage drops back to \( v \) can lead to a worse steady state. Fig. 5 depicts such a case. The untreated economy beginning at \( I \) reaches steady state \( SS \), while the treated economy beginning at \( I \) reaches steady state \( SS' \). Economy \( SS \) has higher output and lower poverty than \( SS' \) and generalized Lorenz dominates \( SS' \).

Without a careful calibration of the model, it is hard to gauge how likely such a scenario is. But, the model does shed light on necessary conditions. This scenario requires \( a < \pi_l / n \), or equivalently,

\[
\pi_h^S < \pi_l^E \pi_l^S / n.
\]

In words, if self-employment returns that are high enough to allow graduation to full-scale entrepreneurship are encountered sufficiently rarely, compared to the frequency of bad outcomes in self-employment and entrepreneurship,\(^{29}\) then micro-credit can make an economy strictly worse off in the long-run.

4.3. Dynamics with saver graduation
Another kind of graduation into the entrepreneurial class may come from saving normal returns rather than achieving supernormal returns. Such “saver” graduation is explicitly ruled out by Assumption (A3). In this section, we assume the reverse of (A3), which guarantees that the savings rate and normal returns in self-employment are jointly high enough to allow for eventual graduation into the upper class. Our goal is

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\(^{28}\) The case where \( \pi_l^E / n < a < 1 / n \) is very similar to the case where \( a > 1 / n \). One difference is that trajectories in the \( v \)-region have slopes less than \( 1 / n \) and thus never exit the region. This implies there is no possibility to raise the wage from \( v \) to \( \tilde{v} \) without micro-credit. Graphically, the whole \( v \)-region would be shaded in Fig. 4. A second difference is that it is possible to exit the \( \tilde{v} \)-region (from its lower left quadrant only) into the \( v \)-region. Apart from these differences, previous results apply.

\(^{29}\) At the same time, the mean self-employment return must allow continuation in self-employment.
assuming that micro-credit can bring development, though sufficient uncertainty in self-employment will not change the result significantly. Differential equations becomes non-linear. However, it seems clear that it appears necessary to track a minimum of five classes, and the system of dynamics when \( \gamma = 0 \), it is clear that \( \gamma P_t^L \) reach the lower middle, but not upper middle, class.

Achieving development from the \( \dot{v} \)-region is now possible. The lower class vanishes with time. The middle class progresses inevitably upward to the upper class. The only drawback is the unlucky entrepreneurs who are dropping into the (lower) middle class.

Now consider the effect of micro-credit. Its main effect will be to move economies that are otherwise destined to remain in the \( \dot{v} \)-region, into the \( \dot{v} \)-region. If the \( \dot{v} \)-region is a transitory step en route to development, then micro-credit can be the economy’s stepping stone to full development.

**Proposition 2.** Under Assumption (A1) and family dynamics as displayed in Fig. 6, if \( \pi^L < n/2 \), micro-credit can bring an underdeveloped (low-wage) economy to full development.

**Proof.** See Appendix.

Fig. 7 displays an economy in the shaded area, \( I \), along with its destination without micro-credit SS and its path to development with micro-credit. Note that micro-credit exists temporarily en route to full-scale, entrepreneurial development. Its ultimate effect is to raise income and lower poverty and inequality.

30 If winner graduation and saver graduation are simultaneously possible, it appears necessary to track a minimum of five classes, and the system of differential equations becomes non-linear. However, it seems clear that allowing some uncertainty in self-employment will not change the result (below) that micro-credit can bring development, though sufficient uncertainty skewed in a negative direction could alter this conclusion.

31 This cutoff \( w^* \) satisfies \( s(w^* + \rho w^*) = w^{**} \).
This result requires that the leakage out of the upper class, \( \pi^E \), be not too large.\(^{33}\) The intuition comes from the steady state distribution under \( \bar{v} \)-dynamics. If in steady state the entrepreneurial class is of measure \( X \), then \( \pi^E X \) will be in the lower middle class and \( \pi^F X \) will be in the upper middle class. Thus the entrepreneurial class measures \( 1/(2\pi^E + 1) \). The condition for this to be greater than the critical value \( 1/(n + 1) \) is that \( \pi^E < n/2 \). More generally, if an unlucky entrepreneur spends \( k \) periods outside of the upper class before regaining enough assets to graduate, the required condition appears to be \( \pi^E < n/k \).

The key condition in this example is that there must not be an income trap at the level of the average endeavor that micro-credit enables. Absent such a trap — that is, if the road to (eventual) wealth accumulation and raised incomes via larger endeavors is open for the average micro-borrower — then micro-credit can be the tool that breaks the initial poverty trap and sets households and the economy on the path to prosperity.

5. Extensions and alternative approaches

Of course, the model cannot pretend to give a final answer to the main question of this paper. It does not deal with some significant issues that bear upon the question. Here we discuss several of these issues.

5.1. Risk aversion

Both this model and the original BN model abstract from risk aversion. If there were positive risk aversion here, market wages would be lowered, since they are the certainty-equivalent values of the occupational payoffs. For example, the middle wage would no longer equal the mean return from self-employment \( \bar{v} \), but something less. Risk aversion would therefore lower the wealth transition paths of wage earners. As a result, the economy might in general have a harder time reaching higher wages and incomes. There may also be no steady states in the \( \bar{v} \)- and/or \( \bar{v} \)-regions.

Micro-credit would have a harder time bringing full development. As shown above, a necessary condition for it to spur full development is that the average return in self-employment allows eventual graduation to the upper class. In a model with risk aversion, the condition would instead be on the certainty equivalent of the self-employment payoff. The intuition, again, is that as the economy approaches the critical labor demand value, most of the middle class is working for wages rather than engaging in self-employment; and if the wage, which equals the certainty equivalent of the self-employment payoff, does not allow graduation, the entrepreneurial class stalls short of its threshold value for development. This makes clear that sufficient risk aversion will make micro-credit unable to catalyze full development even in cases where it could have under risk neutrality.\(^{34}\)

Risk aversion would also open the possibility of additional scenarios in which micro-credit lowers income and raises poverty. By lowering the self-employment certainty-equivalent wage, risk aversion could make exit of the poverty trap hard or impossible for low-wealth households. If so, there may be no steady state in the \( \bar{v} \)-region, and an economy that moves there due to micro-credit would return to the \( v \)-region, potentially with a mix of upper and lower-class agents that leads to lower income and higher poverty in the long-run.

The negative effect of risk aversion could be mitigated, however, to the extent that lenders shoulder the risk of self-employment projects — this would raise the certainty equivalent of the self-employment payoff. It seems quite plausible that lenders are often better positioned than borrowers to bear risk, especially when they have many borrowers and are geographically diversified.\(^{35}\)

5.2. Foreign entry

Entry of foreign entrepreneurs can be examined in this model. The main effect would be a rise in labor demand, which could raise the wage. If foreign entry was sufficient to raise the wage to \( \bar{v} \), the economy would reach maximal income (GNP) with no ill effects; even though there could be fewer domestic entrepreneurs, everyone earns the mean return from entrepreneurship. If it raised the wage to \( \bar{v} \), then poverty would certainly decline but the same long-run equity-efficiency tradeoffs that are potentially involved in micro-credit could surface. Again, the intuition is that the higher wage can diminish the domestic entrepreneurial class in the long-run.

Assume the foreign entrants are permanent upper-class fixtures, rather than facing the downward mobility...
possibilities that domestic entrepreneurs face. In this case, foreign entry can lower income and crowd out domestic entrepreneurship even if it does not affect the wage. It does this (in the \( \nu \)-region) by causing more middle-class agents to work rather than engage in self-employment. Hence fewer people take risk, so fewer graduate into the higher incomes of the entrepreneurial class; fewer drop to the lower class, but income does not drop with this downward transition when the wage is \( \hat{\nu} \).

In summary, sufficient foreign entry of entrepreneurs can bring full development by raising labor demand and wages. If the magnitude of entry is smaller, it can raise the wage moderately and bring the same equity-efficiency tradeoff that micro-credit does. Even if it does not affect the wage, it can lower domestic income and crowd out domestic entrepreneurship.

5.3. “Macro”-credit

By assumption, the introduction of micro-credit lowers \( w^* \) but not \( w^{**} \), that is, it raises access to better technology but not the best. If instead the credit market improvement accesses access to the best technology, entrepreneurship, it can bring full development directly (see footnote 23). But there are also cases in which it can harm an economy, if the increased access to credit is limited.\(^{36}\) For one, it can give rise to the same kind of perverse scenario that micro-credit can when \( a < \pi^c_0/ \nu \) (see Section 4.2.2): raising wages temporarily to \( \hat{\nu} \), but eventually leaving the economy with a less advantageous mix of upper and lower-class agents, lower income and higher poverty.

As another example, assume that entrepreneurship is so risky that entrepreneurs fall to the lowest class when the wage is \( \hat{\nu} \) and they do not succeed. (This would require modification of the second panel of Fig. 2.) Imagine the economy to be in a mid-wage steady state when a credit market improvement broadens access to the entrepreneurial technology. If the entrepreneurial failure rate is high enough,\(^{37}\) the increase in number of entrepreneurs could eventually cause the lower class to grow enough to lower the wage to subsistence level, \( \nu \). The steady state would then likely be in the \( \nu \)-region, with higher poverty and potentially lower aggregate income.\(^{38}\)

In short, “macro”-credit can directly bring development if sufficiently widespread, but if not, can in some instances leave an economy worse off.

5.4. Dynamic incentives

This paper abstracts from the decline in repayment incentives that may occur as borrowers build up their own wealth and lower their need for external finance. However, this issue could be important in answering this paper’s question, since micro-credit would have a difficult time promoting development if it relies on poverty to give incentives for repayment.

The sovereign debt literature has addressed this issue in the context of inter-country debt. The standard assumption is that no direct sanctions for default are available, only the denial of future loans. Making this assumption, Eaton et al. (1986, p. 491) argue and Rosenthal (1991) shows formally that lending for the purpose of capital accumulation (as opposed to consumption smoothing, say) is not possible. The basic intuition is that since transfers to the borrower are concentrated in the early stages of any such relationship, transfers away from the borrower must predominate in later stages — guaranteeing that at some point the borrower will want to default. Anticipating this, the lender will not lend.

This logic suggests that micro-credit cannot facilitate wealth accumulation if repayment is supported only by the threat of future credit denial; it is instead limited to more modest goals, such as aiding consumption smoothing. In contrast, our focus is not on self-enforcing debt contracts but on labor market equilibrium effects of micro-credit. In particular, we follow BN in assuming lenders do have access to direct sanctions \( (P) \), rather than being limited only to withdrawing future credit access; and micro-credit is modeled as a credit market improvement that increases direct sanctions by harnessing social pressure.

5.5. Human capital

The model focuses on physical capital/wealth constraints and ignores human capital. Human capital is clearly a key concern, however. Can poor borrowers graduate to better technologies and greater capital intensity without increases in human capital? Might the increase in wages from micro-credit spur greater human capital investment and catalyze development in this alternative way?

The model can offer some insight into these issues because of the similarities between physical and human capital. For example, in the model micro-credit can raise the wage and allow parents to endow their offspring with

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\(^{36}\) We thank a referee for pointing this out, in particular, the scenario of the next paragraph.

\(^{37}\) Specifically, if \( \pi^b_0 = \pi^b_1 = 0 \), the condition needed is \( \pi^c_0 \geq n/(n+1) \).

\(^{38}\) It would surely be in the \( \nu \)-region if the first panel of Fig. 2 applies and either \( \pi^b_0 = \pi^b_1 = 0 \) or \( a < 1/n \). If failed entrepreneurs transit to a lower class in the \( \nu \)-region and \( \pi^b_0 = \pi^b_1 = 0 \), aggregate income would certainly be lower.
greater physical capital, potentially opening up higher income opportunities; it would presumably have a similar effect in a human capital model. In a simple modification of the model that incorporates both physical and human capital,\textsuperscript{39} we find no qualitative change in the necessary conditions for micro-credit to bring development: average returns in the technology that micro-credit facilitates must be high enough to enable eventual graduation into entrepreneurship through accumulation of physical and human capital. Without this, micro-credit can break the poverty trap but not the mid-wage income trap (for enough people), as in Section 4.2.

That said, there may be differences between human and physical capital that matter for answering this paper’s question. Technological non-convexities and income traps may exist at different levels for the two capitals. In particular, there may be no middle-income trap associated with human capital. Further, the rate of return to human capital could depend in interesting ways on the human capital distribution. Human capital may also interact with technological change. While a fuller analysis of human capital accumulation is beyond the scope of this paper, we believe that some of the insights from our physical capital model would likely carry over.

6. Policy implications and conclusions

Micro-credit in this model reduces poverty in most of the cases we examine. Thus it can indeed bring development, if development is defined as positive impact on the lives of the poorest half of the population (as Muhammad Yunus has advocated).

Can micro-credit bring about the more ambitious goal of high wages and widespread use of efficient technology and capital intensity? This model marks as critical the possibility for saver graduation. That is, graduation must be possible as a rule rather than as an exception – it must be possible to accumulate normal self-employment returns to reach efficient firm scale – if micro-credit is to serve as a stepping stone to long-run development. This possibility depends on two separate quantities, which jointly must be sufficiently high: a) the average return to self-employment activities and b) the savings rate.

Focusing on quantity a) suggests that the productive efficiency of the self-employment undertakings funded by micro-credit institutions is important. This is hardly surprising. It suggests that information sharing, technology transfer, and training programs can provide significant value-added in the long-run.

Micro-credit institutions are also uniquely positioned to affect quantity b). Financial access of all kinds is costly in many contexts in which they operate, due to geographical distances and due to the small amounts in question. In essence, these difficulties can act as a tax on savings. One can show that the savings rate in the presence of a flat-rate tax on savings \( \tau \) would be \( s' = s/(1 - \tau) \). A micro-credit institution can make use of its employee network, village organizational structures (e.g. groups), and geographical extension to offer savings vehicles cost-effectively. This would lower the ‘tax’ and raise the effective savings rate \( s' \), potentially enabling saver graduation.

Thus the model uncovers a novel complementarity between micro-lending and facilitating micro-saving. Not only does savings mobilization raise funds that can be lent (not modeled here), it also enhances the household wealth accumulation process that this model suggests is critical to the long-run success of micro-lending. Here, micro-lending can break the poverty trap, but must rely on micro-saving for help in breaking the mid-wage income trap. A combination of micro-lending and micro-saving facilitation allows for sufficient graduation into entrepreneurship and eventual raising of the wage.\textsuperscript{40}

The model also supports a focus on graduation as a matter of course, which ought to be incorporated into incentives and evaluation of micro-credit programs. Loan officers should be rewarded not only for number of clients and client retention, but also for clients who leave the program for formal financial institutions. On the institutional level, programs might be judged in part on the number of their customers that have moved upward from the micro-credit sector. Information sharing between micro-credit and formal credit institutions would be crucial, enabling graduation to be tracked as well as allowing for mitigation of the default incentives (and adverse selection issues) involved with borrowers on the verge of graduation.

As an alternative to graduation between credit institutions, micro-credit institutions may commit to

\textsuperscript{39} Specifically, we assume that there are three discrete human capital endowments, \( 1 < \sigma < n \), and that each agent receives as part of his bequest the human capital endowment commensurate with his wealth class, lower, middle, or upper. An agent with human capital endowment \( x \) can supply \( x \) units of labor. The only change in the simplex partition (e.g. Fig. 4) is that the boundary dividing the \( \bar{v} \)- and \( v \)-regions intersects the \( P_t = 0 \) line at \( \sigma/(n + \sigma) \) rather than \( 1/(n + 1) \). All the potential scenarios of the model without human capital are present in this model as well.

\textsuperscript{40} Technically, the model has no room for mobilizing saving since all economic activity takes place in an instant. A richer model would allow for saving from the time entrepreneurial returns are realized until bequest takes place, or from the time returns are realized until a new investment opportunity is learned of.
their members indefinitely, providing larger and larger loans and shifting contract terms as needed. It would then be the underlying technological graduation that would be desirable to track and reward. Serving such an array of customers, however, may take most micro-credit institutions beyond their core competencies.

However, the model suggests that graduation is not optional, not simply an added benefit of micro-credit. It is precisely when the rate of graduation is minimal that micro-credit may lower total income and even raise poverty in the long-run. More research on the incentives involved in promoting graduation seems critical.

The model we analyze provides rich ground for examining various micro-credit scenarios that differ in initial conditions, amount of micro-credit, and so on. As with the original BN model, its potential drawbacks include the simplifying assumptions and focus on special, analytically tractable cases. Though we believe there was and is insight to be gained from these cases, the policy implications should be put in that context. Future extensions involving computational analysis may be helpful. Endogenizing the capital and/or labor choices of the self-employed and entrepreneurs, and carrying out simulation with (even roughly) calibrated technology, preference, and uncertainty parameters,\(^{21}\) appear to be fruitful avenues of research.

Appendix A. Proofs of Propositions

Proof of Proposition 1. Relying on the BN Markov assumption, let \(\pi^{SE,v}_k\) be the probability that an agent of class \(j\) transitions to class \(k\) when the wage is \(v\in\{\bar{v}, \hat{v}, \bar{v}\}\) and the agent chooses occupation \(o\in\{SU, L, SE, E\}\), where “SU” indexes subsistence, “L” (wage) labor, “SE” self-employment, and “E” entrepreneurship. Assumption (A3) is reflected as \(\pi_{MH}^{E,\bar{v}}=0\). This of course implies that \(\pi_{HH}^{L,\bar{v}}=0\) and \(\pi_{HH}^{L,\hat{v}}=0\), the latter because \(v<\hat{v}\) by Assumption (A1).

An underdeveloped economy by definition begins with \(P_0^U<1/(n+1)\), and given that micro-credit does not affect \(P_0^U\), an underdeveloped economy affected by micro-credit is either in the \(v\)-region or the \(\bar{v}\)-region. The differential equation describing upper-class evolution in the \(v\)-region is

\[
P_{t+1}^U = \pi_{MH}^{SE,\bar{v}} (1 - (n+1)P_t^U) - (1 - \pi_{HH}^{E,\bar{v}})P_t^U. \tag{8}
\]

The first term captures mobility into the upper class, which includes everyone not working in a firm or running one; wage-earners have no upward mobility by Assumption A3 (\(\pi_{HH}^{L,\bar{v}}, \pi_{MH}^{E,\bar{v}}=0\)). The second term captures mobility out of the upper class. Solving Eq. (8) gives

\[
P_t^U = P_0^U \left(1 - e^{-\left[e_{MH}^{E,\bar{v}}(n+1) + 1 - \pi_{HH}^{E,\bar{v}}\right]t}\right) + P_0^U e^{-\left[e_{MH}^{E,\bar{v}}(n+1) + 1 - \pi_{HH}^{E,\bar{v}}\right]t},
\]

where \(P_0^U\) is equal to \(P_0^U\) defined in Eq. (4), except that \(\pi_{MH}^{SE,\bar{v}}\) is substituted for \(\pi_h^{SE,\bar{v}}\) and \((1 - \pi_{HH}^{E,\bar{v}})\) is substituted for \(\pi_{HH}^{E,\bar{v}}\). This expression makes clear that (i) if \(P_0^U = P_0^U\), then \(P_t^U = P_0^U\) for all \(t\in(0, \infty)\) and (ii) if \(P_0^U \neq P_0^U\), then \(P_t^U \leq \min\{P_0^U, P_0^U\}\) for all \(t\in(0, \infty)\). Since \(P_0^U < 1/(n+1)\) and \(P_0^U \leq 1/(n+1)\) (with equality only when \(\pi_{HH}^{E,\bar{v}}=1\)), we can conclude that \(P_t^U < 1/(n+1)\) for all \(t<\infty\), i.e. that an economy cannot achieve development from the \(v\)-region.

Turning to the \(\bar{v}\)-region, upper-class evolution is described by

\[
P_t^U = \pi_{HH}^{L,\bar{v}} (P_t^L - nP_t^U) + \pi_{MH}^{E,\bar{v}} (1 - P_t^U - P_t^L) - (1 - \pi_{HH}^{E,\bar{v}})P_t^U. \tag{9}
\]

The first term captures mobility into the upper class from the subsisting lower class, whose number includes all the lower class not employed in firms; the wage-earning lower class has no upward mobility by Assumptions (A1) and (A3) (\(\pi_{HH}^{L,\bar{v}}=0\)). The second term captures upward mobility from the middle class, all of whom are self-employed and only the “winners” among whom graduate. The final term captures mobility out of the upper class.

Define \(\epsilon_t\) as the shortfall of the upper class from its critical value for development:

\[
\epsilon_t = \frac{1}{n+1} - P_t^U. 
\]

By hypothesis, \(\epsilon_0 > 0\). Also, \(\dot{\epsilon}_t = -\dot{P}_t^U\), so Eq. (9) can be rewritten

\[
\dot{\epsilon}_t = -\pi_{HH}^{L,\bar{v}} (P_t^L - nP_t^U) + \pi_{MH}^{E,\bar{v}} (1 - P_t^U - P_t^L) + (1 - \pi_{HH}^{E,\bar{v}}) \left(\frac{1}{n+1} - \epsilon_t\right). \tag{10}
\]

\(^{21}\) Caucautt and Kumar (2004) carry out a calibration exercise with the original BN model.
Note that the numbers of subsisters (\(P^U_t - np^U_t\)) and self-employed (\(1 - P^U_t - P^L_t\)) are nonnegative and together sum to \(1 - (n + 1)\delta_t\), which is equivalent to \((n + 1)\epsilon_t\). Given this fact, take two cases. First, consider \(\pi^{E,1}_{j,t}\) < 1. Inspection of Eq. (10) shows that \(\epsilon_t\) > 0 when \(\epsilon_t\) is close enough to zero. Thus \(\epsilon_t\) is bounded away from zero. Second, consider \(\pi^{E,1}_{j,t} = 1\) and define \(\pi\) as the maximum of \(\{\pi^{E,1}_{j,t}, \pi^{E,1}_{j,t}\}\). Then

\[
\dot{\epsilon}_t \geq -\pi(P_t^n - nP^U_t - \pi(1 - P^U_t - P^L_t)
\]

\[
= -\pi(n + 1)\epsilon_t,
\]

from which it follows that

\[
\epsilon_t \geq \epsilon_0 e^{-\pi(n + 1)t}.
\]

Thus, \(\epsilon_t\) is bounded away from zero for any finite \(t\). This establishes that development is not attainable from the \(\nu\)-region.

Proof of Proposition 2. Sufficient micro-credit guarantees a wage of \(\hat{v}\) since it can lift enough of the lower class to guarantee that \(P^L_t \leq nP^U_t\). Assumption (A1) and the family dynamics of Fig. 6 give rise to the system of equations described by \(P^U_t = P^U_0 e^{-\pi t}\) and differential Eq. (7), where \(v_t = \hat{v}\). Solving gives

\[
P^U_t = P^U_{SS}
\]

\[
- e^{-(1 + \pi^E/2)t)} \left[ \delta^U \cos(\beta t) + \frac{\delta^UM + (1 - \pi^E)\delta^U_0}{\beta} \sin(\beta t) \right]
\]

\[
- \frac{\gamma P^U_0 e^{-\pi t}}{\pi^E} \left[ 1 - e^{-\pi^E/2} \left( \cos(\beta t) + \frac{\pi^E}{2\beta} \sin(\beta t) \right) \right],
\]

(11)

where \(P^U_{SS} = \pi^E / (2\pi^E + 1), P^U_t = 1 / (2\pi^E + 1), \delta^U = P^U_t - P^U_0, \delta^UM = P^U_{SS} - P^U_0, \beta = \sqrt{(4 - \pi^E)/2}\). It is clear from Eq. (11) that \(P^U_t\) approaches \(P^U_{SS}\) as \(t\) gets large. Manipulation of the \(P^U_{SS}\) expression gives that \(P^U_{SS} > 1 / (n + 1)\) if \(\pi^E < n/2\). This condition then guarantees that under \(\hat{v}\)-dynamics, \(P^U_t\) will reach the critical value \(1 / (n + 1)\) in finite time and \(\hat{v}\)-dynamics will take over. Eq. (3) guarantees that when \(v_t = \hat{v}\), \(P^U_t\) will monotonically increase toward its steady state value of \((n + 1 - \pi^E) / (n + 1)\).

It remains to show that the trajectory through the \(\dot{\nu}\)-region never crosses into the \(\nu\)-region; if it did, the dynamics would change and the above analysis would no longer apply. Trajectories in the \(\dot{\nu}\)-region satisfy \(P^U_t = P^L_t\). If \(P^U_t = 0\), it is impossible to enter the \(\nu\)-region, that is, to satisfy \(P^U_t > nP^U_t\); assume \(P^U_t > 0\). Since \(P^U_t\) is declining, to exit the \(\nu\)-region requires that \(P^U_t < 0\) and that \(\gamma P^U_t > 1 / n\). Note from Eq. (7) that \(P^U_t > P^L_t\). Combining this with \(P^U_t = P^L_t\) gives that \(\gamma P^U_t < 1 / n\) (when \(P^U_t > 0\)). Now consider a point in the interior of the \(\dot{\nu}\)-region near the boundary with the \(\nu\)-region, the line \(P^U_t = nP^U_t\). This point can be expressed as \((P^U_t, P^U_t) = (nk, \lambda k),\) where \(k \in [v, 1]\) and \(\lambda \in [1, 1/\pi^E]\). We know that here \(\gamma P^U_t \leq \pi^E/2\) (if \(P^U_t < 0\), which is less than \(1/n\) for \(\lambda\) satisfying \(1 < \lambda < 1/\pi^E\)). This establishes that economies close to the boundary (\(\lambda\) near 1) are not following trajectories that allow for exit from the \(\dot{\nu}\)-region into the \(\nu\)-region; that is, exit downward is impossible.

**References**


