Comment on “A note on the Hotelling principle of minimum
differentiation: Imitation and crowd”

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Abstract

We argue that the minimum differentiation firm location equilibrium and the pure strategy pricing equilibrium in Di Cintio’s [Di Cintio, M., 2007. A note on the Hotelling principle of minimum differentiation: Imitation and crowd. Research in Economics 61 (3), 122–129] “Note” need not exist under the conditions claimed.

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Di Cintio (2007) argues that the introduction of quadratic consumption externalities to the Hotelling (1929) model of spatial competition restores continuity to firm demand and guarantees the existence and uniqueness of the location equilibrium where firms locate at the mid-point of Hotelling’s line. This finding is of potentially great significance; if true, it shows what a small and plausible modification is needed to restore validity to Hotelling’s principle of minimum differentiation, an idea that has been dubious since d’Aspremont et al. (1979) and Osborne and Pitchik (1987).

However, the analysis is incomplete.

Throughout this comment, for ease of exposition we let \( k = -C_{\text{ext}}(N) \) be the externality cost a consumer experiences when purchasing from the same firm as everyone else. Di Cintio (2007) assumes that \( k > 0 \) and (implicitly) that the transportation cost \( t \) satisfies \( t > 0 \).

Consider first Proposition 2, which claims existence and uniqueness of the location equilibrium where both firms locate on the mid-point of Hotelling’s line \( (x_1 = x_2 = 1/2) \) and charge \( p_1^* = p_2^* = t + k \). Consider a deviation by firm 1 to price \( p_1 = t - q \), for some \( q \in (0, t) \). We argue that with this deviation, firm 1 captures the whole market.\(^1\) This is apparent since a consumer would pay \( p_2 = t + k \) at firm 2, or at firm 1, \( p_1 = t - q \) as the direct price plus \( k \) in congestion costs (since every other consumer makes the same choice). Hence, all consumers prefer firm 1 with its maximum total price of \( t + k - q \). (We can ignore transportation costs here since it costs each consumer the same to travel to either firm.) Thus, with this deviation firm 1 captures the market and earns profits of \( (t - q)N/2 \). Clearly, if \( k < t \), any deviation involving \( 0 < q < (t - k)/2 \) is strictly profitable. This provides a counterexample to Proposition 2, which asserts the existence and uniqueness of the stated equilibrium with no conditions on \( k \) or \( t \) (except that they be positive).

\(^1\) It would not according to Eqs. (5), (6) and (6'), but we argue below that these are not universally valid.
The argument for Proposition 2 follows from the claim in Proposition 1 of a unique pure strategy pricing equilibrium in the pricing stage for a wide set of firm locations. But, there are also counterexamples to Proposition 1. The above example where \( x_1 = x_2 = 1/2 \) and \( p_1^* = p_2^* = t + k \) is one. A bit more generally, consider any firm locations symmetric about the mid-point (i.e., satisfying \( x_1 + x_2 = 1 \)) with inter-firm distance \( \delta \equiv x_2 - x_1 \) strictly less than 1/2.\(^2\) Proposition 1 guarantees \( p_1^* = p_2^* = t + k \) as the unique pure strategy pricing equilibrium. We argue that firm 1 can capture the market with price \( p_1 = t - q - t\delta \) for some \( q \) satisfying \( 0 < q < t(1 - \delta) \). This is apparent since a consumer would pay \( p_2 = t + k \) at firm 2, or at firm 1, \( p_1 = t - q - t\delta \) as the direct price plus \( k \) in congestion costs (since every other consumer makes the same choice) plus a maximum transportation cost differential of \( t\delta \) (the cost of going from one firm to the other). Hence, all consumers prefer firm 1 with its maximum total price of \( t + k - q \). With this deviation firm 1 captures the market and earns profits of \( [t(1 - \delta) - q]N \). Equilibrium profits are \( (t+k)N/2 \).

If \( k < t(1 - 2\delta) \), any deviation involving \( 0 < q < [(t(1 - 2\delta) - k)/2 \) is strictly profitable and, thus, the equilibrium \( (p_1^*, p_2^*) \) does not exist.

The above counterexamples highlight the idea that introducing a small congestion cost may not solve Hotelling’s non-existence problem. If \( k \) is small, it will have only a small impact on the prices that capture the market and, thus, on firms’ incentives to undercut each other. Hence, a small \( k \) can do little to restore the existence of a pure strategy pricing equilibrium at most locations.

Finally, we argue that the main shortcoming in the analysis supporting Propositions 1 and 2 is over-reliance on Eq. (5). This equation – along with Eqs. (6)–(10), which are derived directly from Eq. (5) – is used to characterize demand. Eq. (5) does so by providing an equilibrium division of consumers between firms 1 and 2 defined by indifference point \( \hat{x} \), such that all consumers to the left of point \( \hat{x} \) patronize firm 1 and all consumers to the right of point \( \hat{x} \) patronize firm 2. But, algebraically, Eq. (5) is a valid expression for the indifference point \( \hat{x} \) only when \( \hat{x} \) falls between the two firms, because it solves indifference Eq. (3) assuming that \( x_1 \leq \hat{x} \leq x_2 \) (which allows resolution of the absolute value operators in (3) in a specific way).

Thus, expression (5) is only valid when \( \hat{x} \in [x_1, x_2] \), but the analysis assumes that it is valid whenever \( \hat{x} \in [0, 1] \). This error leads to demand being misspecified for certain ranges of prices at most firm locations; and the demand misspecification then feeds into profit functions and best responses, and ultimately to the results of the propositions.

In summary, the propositions in Di Cintio (2007) do not hold as written, and this appears to stem from a misspecification of demand. See Ahlin and Ahlin (2006) for independent analysis in a very similar setting.\(^3\) The conclusions turn out to be similar, but not as stark. In particular, they find that for any firm locations \( x_1 < 1/2 < x_2 \), high enough congestion costs relative to transportation costs \( (k/t) \) guarantee existence and uniqueness of the Hotelling pricing equilibrium. This result is then used to show that the maximum differentiation between firms in any asymmetric, pure strategy location equilibrium is declining in \( k/t \), and approaching zero. Thus, Hotelling’s principle of minimum differentiation is more valid the higher are congestion costs relative to transportation costs (\( k/t \)), and becomes arbitrarily valid as \( k/t \) gets large.

References


\(^2\) d’Aspremont et al. (1979) establish that at these locations, no pure strategy pricing equilibrium exists (in the absence of consumption externalities).

\(^3\) Ahlin and Ahlin (2006) use a negative, linear consumption externality: \( -\alpha N \), where \( \alpha > 0 \), which differs from the quadratic externality of Grilo et al. (2001) used by Di Cintio (2007): \( aN_1 - \beta N_2^2 \), where \( a, \beta > 0 \). But, the two are isomorphic, given that the two firms split a market of size \( N \). Under linearity, the differential externality costs incurred by patronizing store 1 rather than 2 are \( \alpha N_1 - \kappa N_2 \), which, since \( N_2 = N - N_1 \), equals \( \kappa (2N_1 - N) \). Under the quadratic specification, the externality cost differential for patronizing store 1 instead of store 2 is \( (\beta N_1 - \alpha)(2N_1 - N) \).

The cost differentials are linear under both specifications, and identical for \( \kappa = \beta N - \alpha \). (Note that \( \beta N - \alpha \) is strictly positive under Di Cintio’s assumption that \( C^{\text{ext}}(N) = aN - \beta N^2 < 0 \).) Since cost differentials are all that matter when consumers compare the two firms, this proves that for fixed \( N \), competition under linear congestion costs can be mapped into competition under quadratic costs of the form used here, and vice versa.