Methods Exam Questions, May 2013

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Microeconomics (Part I)

(Remark: Answer all questions. All questions have several parts and some questions have hints on how to approach them. So read carefully before you start writing)

1. (Production and Firm Behavior) Consider the following cost function of a competitive firm:

\[ C(w_x, w_y, w_z; q_1, q_2) = 2\sqrt{w_x w_y q_1 + w_x w_z (q_2)^2} \]

where \( w_x, w_y, w_z \geq 0 \) are the prices of inputs \( x, y \) and \( z \), respectively; \( q_1, q_2 \geq 0 \) are the quantities of the first and the second final good, respectively.

(a) Show that the cost function is homogeneous of degree one in the input prices.
(b) Find the compensated demands for inputs \( x, y \) and \( z \).
(c) Find the production function \( f: \mathbb{R}^3_+ \rightarrow \mathbb{R}^2_+ \) that transforms inputs \( x, y \) and \( z \) into final goods \( q_1 \) and \( q_2 \).
(d) Suppose the price of the first final good is zero. Find the competitive profit maximizing supply of good 2.
(e) Suppose the price of the second final good is zero. Find the competitive profit maximizing supply of good 1.

2. (General Equilibrium with Externality) Consider an economy with two private goods, \( x \) and \( y \), and two individuals Mary and Bob. When Mary consumes \( x_m \) it creates an externality for Bob, and when Bob consumes \( x_b \), it creates an externality for Mary. Preferences are

\[ U_m(x_m, y_m, x_b) = y_m + \alpha_m \ln x_m - \beta_m \ln x_b \]

and

\[ U_m(x_b, y_b, x_m) = y_b + \alpha_b \ln x_b - \beta_b \ln x_m \]

where the subscripts \( b \) and \( m \) refer to Bob and Mary, respectively. Each person is endowed with \( R \) units of good \( y \), and \( y \) is an input into the production of \( x \). One unit of \( y \) produces one unit of \( x \).

(a) Find the competitive equilibrium. Is this equilibrium Pareto efficient? Prove your claim.
(b) Suppose that \( x_m \) and \( x_b \) are observed by the government and can be taxed at different rates. Find the optimal tax rates.
(c) Now suppose that the government cannot observe \( x_m \) and \( x_b \) but can tax production of good \( x \) at a constant rate. Find the optimal tax rate.
(d) Show that the tax rate solved for in part (c) is a weighted average of those derived in part (b). Explain.

3. (Auction) A plot of land is auctioned off to two bidders, A and B, who intend to use it for oil extraction. The plot is divided into two parts, the northern part and the southern part. The amount of oil contained in the northern part is valued at \(v_N\) and in the southern part at \(v_S\) (so the entire plot contains \(v_N + v_S\) worth of oil). Both \(v_S\) and \(v_N\) are random variables that are distributed uniformly on \([0,1]\) and are independent of each other.

The auction format is the sealed-bid second-price auction (the bidders simultaneously submit nonnegative bids \(b_A\) and \(b_B\); the highest bidder wins the entire plot and pays the second-highest bid; ties are resolved by tossing a fair coin). The two parts of the plot cannot be auctioned off separately. Both bidders value the plot at \(v_N + v_S\) and are risk neutral.

(a) Suppose that bidder A observes the realization of \(v_N\), but not of \(v_S\), and bidder B observes the realization of \(v_S\), but not of \(v_N\). Then the second-price auction can be represented as a Bayesian game where the type of bidder A is \(v_N\) and the type of bidder B is \(v_S\).

(i) In this game, define a pure strategy for a bidder.

(ii) Write down the expected payoff of bidder A of type \(v_N\) who has submitted a bid \(b_A\), given that bidder B follows a strategy of the form \(b_B(v_S) = \alpha v_S\), where \(\alpha > 0\).

(iii) This game has a symmetric Bayesian-Nash equilibrium of the form:

\[
\begin{align*}
    b_A(v_N) &= \alpha v_N; \\
    b_B(v_S) &= \alpha v_S
\end{align*}
\]

where \(b_i(\cdot)\) is the strategy of bidder \(i\) and \(\alpha > 0\). Find \(\alpha\).

(b) Now suppose that bidder A observes the realizations of both \(v_N\) and \(v_S\), but bidder B observes only the realization of \(v_S\). Consider the following strategy profile:

\[
\begin{align*}
    b_A(v_N, v_S) &= v_N + v_S; \\
    b_B(v_S) &= v_S + \frac{1}{2}
\end{align*}
\]

Is this strategy profile a Bayesian-Nash equilibrium? Prove your answer.

(c) Suppose bidder A has observed that \(v_N = \frac{1}{3}\). He is offered the following option: for the price of \(p \geq 0\), he can also observe the realization of \(v_S\). He knows that if he chooses to observe \(v_S\), the strategy profile described in (b) will be played; if he declines, the Bayesian-Nash equilibrium described in (a) will be played. What is the highest price \(p\) that bidder A is willing to pay for the option to observe \(v_S\)?

4. (Monopolistic Screening) Consider the following variation of the canonical monopolistic screening problem: (i) A monopolist has one unit of the good for sale. (ii) Cost of production is zero. (iii) The Buyers have private valuation for the good—the valuation can be either high, \(\theta_H\), or low, \(\theta_L\), where \(\Pr(\theta_H) = \lambda\) and \(\theta_H - \theta_L > 1\). Moreover, assume that
the buyer’s payoff function changes with his type: if the monopolist charges $T$ for the good, a low type buyer’s payoff from buying the good is:

$$\theta_L - T,$$

whereas the payoff of the high type is:

$$\log(\theta_H - T).$$

Note that this payoff structure suggests that the buyer’s risk aversion rises with her valuation. Suppose that both types of the buyer have an outside option of 0.

(a) If $\theta$ is public information, what is the first-best allocation? That is, what price would the monopolist charge to each type? Would both types buy the good?

(b) Show that the seller can implement the first-best outcome (that is, the allocation you have derived in part (a) above) by using a random scheme [Hint: The menu options are of the form (i) sell for sure at price $T^*$ and (ii) sell for sure either at price $T_1$ with probability $\alpha$ or at price $T_2$ with probability $1 - \alpha$. You need to find what the values of $T$s and $\alpha$ should be such that the first-best is achieved—every type who was buying the good under first-best, continues to buy the good (and the type(s) who did not buy, if any, still do(es) not buy) and both types have the same utility in expectation.]